CS 275 001/002 Exam 4 April 19, 2007 Solutions

There are a total of 87 points (12, 16, 15, 8, 16, 10, and 10) for this exam.

1. This problem is easier answered out of order.
(c) Remember that we are constructing a weighted tree, not a shortest weighted path. Hence, we have two choices at a minimum for the root of the minimum spanning tree. This means the tree cannot be unique.

- 2 no answer
- 1 wrong answer or no explanatio

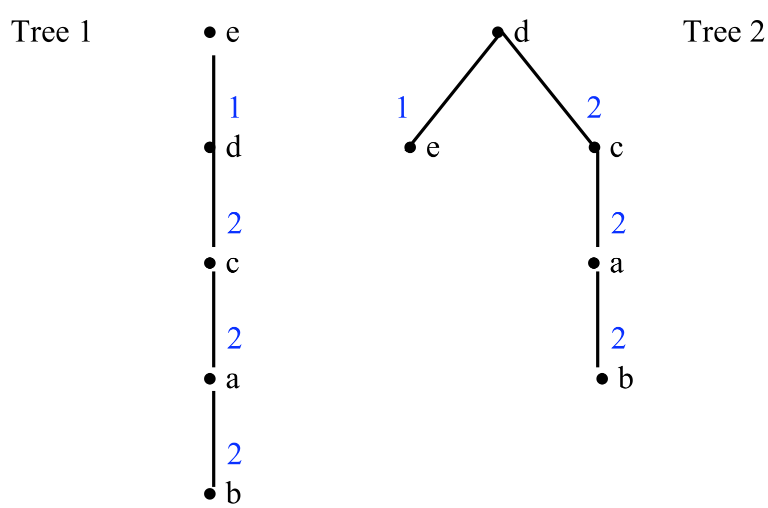
(b) There are two algorithms in the book and class notes: Pim's and Kruskal's. Either works fine and can produce two different minimum spanning trees.

- 2 no answer
- 1 wrong answer

(a) The only possible steps of Pim's and Kruskal's algorithm are the following:

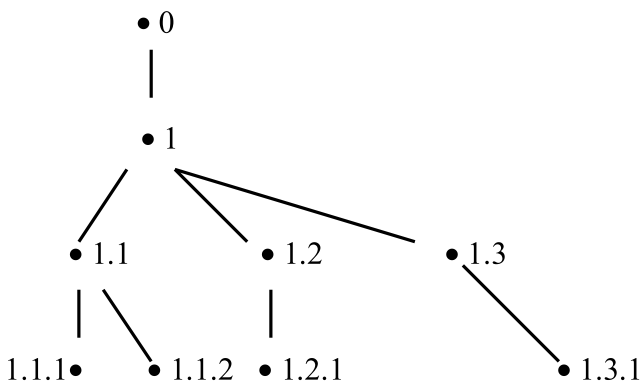
Tree 1 Steps	Tree 2 Steps
Add {e,d}	Add {d,e}
Add {d,c}	Add {d,c}
Add {c,a}	Add {c,a}
Add {a,b}	Add {a,b}

It is not unique since there is a choice of either vertex d or e as the root of the minimum spanning tree. The two possible trees look like



- 8 no answer
- 4 did not follow stated algorithm
- 3 wrong tree constructed
- 2 constructed a shortest pat, not a tree

2. (a)



- 4 no answer
- 2 missed a level

(b) preorder: 0, 1, 1.1, 1.1.1, 1.1.2, 1.2, 1.2.1, 1.3, 1.3.1
(c) inorder: 1.1.1, 1.1, 1.1.2, 1, 1.2.1, 1.2, 1.3.1, 1.3, 0
(d) postorder: 1.1.1, 1.1.2, 1.1, 1.2.1, 1.2, 1.3.1, 1.3, 1, 0

- 4 no answer each (b)-(d)
- 1 per mistake (b)-(d)

3. This problem had some real doozies for answers. You were either graded by part or overall. For overall,

- 15 no answer
- 7 all parts are too vague

(a) I was looking for graph coloring, though picking a graph format was accepted in this part if it actually could be useful in parts (b) and (c).

- 4 no answer
- 2 four color theorem stated
- 2 too vague

(b) Each species is a vertex. Draw edges between any species where either could eat the other. Then apply graph coloring to the vertices to get the minimum number of compounds.

- 6 no answer
- 3 too vague
- 3 not going to produce the minimum number of compounds
- 3 close, nut not close enough
- 3 four color theorem used

(c) The chromatic number produced in (b) is the minimum number of compounds. You cannot pick a set number of compounds since the graph in (b) is not a planar graph (hence, the four color theorem does not apply). To see this, consider n species, n > 4, that eat all of the other species. Then clearly n compounds are necessary, which is the maximum number of compounds.

- 5 no answer
- 3 four color theorem used
- 2 too vague
- 2 close, but not close enough

4. There are two sets of paths: those that end with the {d,b} edge and those that end with the {c,b}. The {d,b} paths pass through vertices afdb, aedb, and either (but not both) aefdb or afedb (since edge {e,f} is also edge {f,e}). The {c,b} paths pass through afcb, afccb, aefcb, aefccb, aedfcb, and aedfccb. Hence, there are 9 simple paths from a to b.

- 8 no answer
- 4 missing paths
- 2 missed loop at vertex c
- 1 duplicate path
- 1 non-simple path

5. For me, the easiest way to check these relations is to construct the adjancy matrices and inspect them by eye. You can also go to the basic definitions.
(a) reflexive, antisymmetric, and transitive
(b) symmetric
(c) just a relation
(d) reflexive and symmetric

- 16 no answer to whole problem
- 4 maximum per part (a)-(d)
- 1 for each wrong within a part

6. There were lots of interesting 4×4 adjacency matrices. Some of them were equivalence relations, too.

- 10 no answer to whole problem
- 6 trivial adjacency matrix
- 5 not an equivalence relation
- 4 no digraph

7. By definition, $[i,i+1) = \{ x \in \mathbf{R} \mid i \le x < i+1 \} \neq \varnothing$. It is immediate that $A_i \cap A_j = \varnothing$ if $i \neq j$ for all $i,j \in \mathbf{Z}$. It is also immediate that $\bigcup_{i \in Z} A_i = \mathbf{R}$. Hence, we have a partition of **R**.

- 10 no answer
- 10 stated false and proof completely false
- 9 stated true and no proof
- 7 stated false and proof somewhat false
- 6 stated true and proof a fishing expedition
- 5 stated true and proof completely false
- 5 correct proof for the wrong sets
- 5 stated false and just missed that $A_i \subset \mathbf{R}$