

CS 275 Sections 001 and 002 Exam 3 March 29, 2007

You may use the class notes from the course web site, your own notes, and the textbook only for references during the exam. You have approximately 70-75 minutes.

Write your name and the page number on each piece of paper you use. If you use the back, label that, too. If you split a problem's solution, please note this fact. Show your work. There are 5 problems on one side/page. The extra credit problem will decrease your point total of all of your exams for the semester. Do not miss it!

No electronic devices are to be used. Turn off your cell phone now. Answering one means you are done with your exam on the spot.

1. (a) Explain why the generating function for the number of ways to have n cents in pennies and nickels is $(1 + x + x^2 + x^3 + \dots)(1 + x^5 + x^{10} + x^{15} + \dots)$.
(b) Find the generating function for the number of ways to have n cents in pennies, nickels, dimes, and quarters.
2. Find the generating function for $0, 0, 1, a, a^2, a^3, a^4, \dots$ for $a \neq 0$.
3. Find the recurrence relation and initial condition that uniquely determines each of the geometric progressions
(a) $2, 10, 50, 250, \dots$
(b) $6, -18, 54, -162, \dots$
4. Solve the recurrence relation $2a_n - 11a_{n-1} + 5a_{n-2} = 0, n \geq 2, a_0 = 1, a_1 = -8$.
5. Let $f(n) = 5f(n/2) + 3$ with $f(1) = 7$.
(a) Find $f(2^k)$ for any $k \in \mathbf{N}$.
(b) Assume f is an increasing function. Estimate $f(n)$.

Extra credit: In the class notes on page 156 is the following example:

I hand back graded exams randomly. What is the probability that no student gets his or her own exam? It is $P_n = D_n / n!$ since there are $n!$ possible permutations. As $n \rightarrow \infty$, $P_n \rightarrow e^{-1}$.

Prove that e^{-1} is the correct answer.