CS 275 001/002 Exam 3 March 29, 2007 Solutions

There are a total of 73 points (5, 7, 10, 6, 6, 8, 10, 9, 11) for this exam.

1. (a) The generating function for pennies is $\sum_{k=0}^{1} x^k$ whereas the generating function for nickels is $\sum_{k=0}^{1} x^{5k}$.

   10 no answer
    3 wrong explanation for pennies
    3 wrong explanation for nickels
    2 no explanation for multiplication

   (b) Extrapolate part (a) to get factors with powers of 1, 5, 10, and 25.

   10 no answer
    5 wrong answer: (1+x+x^2+....).(1+x^5+x^6+x^7+...).(1+x^10+x^11+..)
    5 wrong answer: 1 + (1+x^5+x^10+x^25) + (1+x^5+x^10+x^25)^2 + ...

2. $x^2 / (1-ax)$.

   10 no answer
    6 wrong summation
    2 incorrect closed form solution
    0 no closed form solution

3. (a) $a_n = 5a_{n-1}$, $n \geq 1$, $a_0 = 2$.

   10 no answer
    3 assume an intial a_0, so had wrong initial conditions
    6 a regular function instead of a recurrence relation

   (b) $a_n = -3a_{n-1}$, $n \geq 1$, $a_0 = 6$.

   10 no answer
    3 assume an intial a_0, so had wrong initial conditions
    6 a regular function instead of a recurrence relation

4. $a_n = (26/9)(1/2)^n - (17/9)(5)^n$, $n \geq 0$.

   10 no answer
    6 correct characteristic equation, but failed to find roots
    3 found roots correctly, but did not solve for alpha1 and alpha2 correctly
    3 followed the method correctly, but had many problems with arithmetic

5. The easy way is to cite Example 6 on page 429 of the textbook (5^{th} ed.) or you can do it the hard way by providing a proof:

(a) Use the master divide and conquer theorem. Then for $n = 2^k$,

$$f(n) = 5^k[f(1) + (3 / (5 - 1)] + (-3 / (5 - 1)$$
$$= 5^k[7 + (3/4)] - (3/4)$$
$$= 7.75 \times 5^k - .75.$$

10 no answer
5 some errors
0 just cited example in book and did no more (my favorite answer)

(b) The master divide and conquer theorem states that f(n) is $O(n^{\log_2 5})$.

10 no answer
6 no O notation and expression wrong
5 O notation and just wrong
1 no O notation and expression right for f(n)

Extra credit: We know that

$$D_n = \left(1 - \sum_{k=1}^{n} (-1)^k \frac{1}{k!}\right) n!$$

Hence, $P_n / n! = \sum_{k=0}^{n} (-1)^k \frac{1}{k!} = e^{-1}$ from Table 1 on page 440 of the textbook (5^{th} ed.) and page 150 of the class notes (the series for e^x with $x = -1$).

-6 correct proof
0 otherwise