

CS 275 001/002 Exam 2 March 1, 2007 Solutions

There are a total of 73 points (5, 7, 10, 6, 6, 8, 10, 9, 11) for this exam.

1. Consider any element of $A+B$: $a_{ij} + b_{ij} = b_{ij} + a_{ij}$, which is any element of $B+A$. We used the commutative property of scalar arithmetic.
 - 5 pts: no answer
 - 4 pts: picked specific one A and B and proved result for just that example
 - 3 pts: wrong definition of $A+B$ or $B+A$
 - 2 pts: not using arbitrary (i,j) element or proved for a specific m and n
 - 2 pts: used something other than scalar arithmetic commutative property
 - 1 pt: commutative property of scalar arithmetic insufficiently stated

2. (a) The 2×2 identity matrix.
 - 4 pts: no answer
 - 1 pt: each wrong entry in product
 (b) inverses
 - 3 pts: no or wrong answer

3. (a) $H_1 = 1$, $H_k = H_{k-1} + k^{-1}$, $f(0) = 0$, $f(n) = f(n-1) + n^{-1}$, or similar.
 - 2 pts: no answer
 - 1 pt: answer not in terms of previous terms
 (b) The basis is $\sum_{k=1}^1 H_k = H_1 = 1 = 2 \times 1 - 1 = (1+1)H_1 - 1$. Now assume the truth of $\sum_{j=1}^k H_j = (k+1)H_k - k$. This assumption leads us to

$$\begin{aligned}
 \sum_{j=1}^{k+1} H_j &= \sum_{j=1}^k H_j + H_{k+1} \\
 &= (k+1)H_k - k + H_{k+1} \\
 &= (k+1)[H_k - (k+1)^{-1}] - k + H_{k+1} \\
 &= (k+2)H_{k+1} - 1 - k \\
 &= (k+2)H_{k+1} - (k+1)
 \end{aligned}$$
 - 7 pts: no answer
 - 4 pts: proving wrong problem (e.g., missed the sum on left hand side) or not even close
 - 2 pts: on right track, got slightly derailed at end
 (c) Vanilla mathematical induction.
 - 1 pt: wrong form of induction or no proof in (b)

4. (a) $288 = 4 \times 12 \times 3 \times 2$
 3 pts: no answer
 1 pt: wrong or no product
 (b) $24 = 4 \times 1 \times 3 \times 2$
 3 pts: no answer
 1 pt: wrong or no product
5. (a) $8! = 40320$
 2 pts: no or wrong answer
 (b) $7! = 5040$
 2 pts: no or wrong answer
 (c) $6! = 720$
 2 pts: no or wrong answer
6. (a) 1
 2 pts: no answer
 1 pt: not evaluated
 (b) 120
 2 pts: no answer
 1 pt: not evaluated
 (c) 120 (b) = (c)
 2 pts: no answer
 1 pt: not evaluated
 (d) 7260
 2 pts: no answer
 1 pt: not evaluated
7. (a) $C(5,3) = 10$
 5 pts: no answer
 3 pts: wrong answer
 1 pt: not evaluated to answer
 (b) $C(5,4) = 5$
 5 pts: no answer
 3 pts: wrong answer
 1 pt: not evaluated to answer
8. (a) $D_1(1) = D_1(2) = D_1(3) = D_1(4) = D_1(5) = D_1(6) = 1$,
 $D_2(1) = D_2(2) = D_2(3) = D_2(5) = D_2(6) = 1$, $D_2(4) = 2$, and
 $D_3(1) = D_3(3) = D_3(4) = D_3(5) = D_3(6) = 1$, $D_3(2) = 3$.
 2 pts each: no answer
 1 pt each: wrong answer
 (b) $P(D_1 = i) = 1/6$,
 $P(D_2 = i) = 1/7$, $1 \leq (i \neq 4) \leq 6$, and $P(D_2 = 4) = 2/7$, and
 $P(D_3 = i) = 1/8$, $1 \leq (i \neq 2) \leq 6$, and $P(D_3 = 2) = 3/8$.
 1 pt each: wrong or no probability

9. (a) Binomial distribution.
 2 pts: no answer, no distribution name, or just plain wrong
- (b) Laptop either works or not after every update (though I might not notice).
 2 pts: no or flagrantly wrong answer
 1 pt: only somewhat wrong answer
- (c) Let $p = .95$ and $q = .05$. The formula is $C(k,k-1)p^{k-1}q = kp^{k-1}q$.
 5 pts: no answer
 3 pts: incomplete
 2 pts: wrong answer based on your (a) and (b)
- (d) We want $0.5 \cong k \times (.95)^{k-1} \times .05$. The maximum of this function is slightly less than 0.4, so we never have a 50% chance.
 2 pts: no answer
 1 pt: no estimate based on your (c)

