

CS 275 Sections 001 and 002 Exam 1 February 1, 2007

You may use the class notes from the course web site, your own notes, and the textbook only for references during the exam. You have approximately 70-75 minutes.

*No electronic devices are to be used. Turn off your cell phone now. Answering one means you are done with your exam on the spot.*

1. Let  $p$  and  $q$  be logic propositions such that  $p \rightarrow q = F$ . Determine the truth values for the following:
  - a.  $p \wedge q$
  - b.  $\neg p \vee q$
  - c.  $q \rightarrow p$
  - d.  $\neg q \rightarrow \neg p$
  
2. Determine *all* T and F values for the logic propositions  $p$ ,  $q$ ,  $r$ ,  $s$ , and  $t$  such that the following are F (you do not need a 16 case truth table to complete this problem):
  - a.  $((p \wedge q) \wedge r) \rightarrow (s \vee t)$
  - b.  $((p \wedge q) \wedge r) \rightarrow (s \oplus t)$
  
3. Using the well known logical inference laws, e.g., Identity, Domination, Idempotent, Negation, Double Negation, Commutative, Associative, Distributive, DeMorgan, and Absorption (pp. 14-15 in the class notes and a Table in the textbook), simplify the logic proposition  $(p \vee q) \wedge \neg(\neg p \wedge q)$  as much as possible and identify which laws you use per step.
  
4. Consider each of the following arguments. If the argument is valid, identify the rule of inference that establishes its validity. If not, indicate whether the error is due to an attempt to argue by the converse or inverse.
  - a. Joe can program in C++ and also in Fortran. Therefore Joe can program in C++.
  - b. A sufficient condition for Jan to win the golf tournament is that Jorge not sink a birdie on the last hole. Jan won the tournament. Therefore, Jorge did not sink a birdie on the last hole.
  - c. If Meg's computer program is correct, then she will be able to complete her assignment in at most one hour. It takes Meg more than one hour to complete the assignment. Therefore, Meg's computer program is incorrect.
  - d. Bill's car keys are either locked in the car or on the kitchen counter. Bill's car keys are not on the kitchen counter. Therefore, Bill's keys are locked in the car.
  - e. If interest rates fall, then the stock market will rise. Interest rates are not falling. Therefore, the stock market will not rise.

5. Use the Chinese Remainder Theorem to tell me how many things I have if that number, when divided by 11 has remainder 4 and when divided by 7 has remainder 5. Show all of your work.
6. The following parts relate to prime numbers, relative primes, and linear congruences:
  - a. Factor 22, 71, and 105 using the Fundamental Theorem of Arithmetic.
  - b. Show if
    - i.  $34 \equiv 5 \pmod{4}$
    - ii.  $34 \equiv 4 \pmod{2}$
  - c. Without factoring 256, could 17 be one of its prime factors? State a theorem or lemma from the textbook or class notes justifying your answer (cite the source, textbook or class notes, and page number).
7. Consider the algorithm Foo defined below. Using Big-Oh notation,  $O(\dots)$ , what is the complexity of algorithm Foo? Show details.

```
algorithm Foo(n: integer  $\geq 2$ )
  for i := 1 to n-1
    for j := 1 to 15
      ... (simple operations of  $O(1)$ )
  for i := 1 to n
    for j := 1 to n/2
      ... (simple operations of  $O(1)$ )
  {Algorithm results here}
```