CS 275 001/002 Exam 1 February 1, 2007 Solutions

1. Let p and q be logic propositions such that $p \rightarrow q = F$. Determine the truth values for the following:
    a. $p \wedge q$
    b. $\neg p \vee q$
    c. $q \rightarrow p$
    d. $\neg q \rightarrow \neg p$

p	q	$p \rightarrow q = F$	$p \wedge q$	$\neg p \vee q$	$q \rightarrow p$	$\neg q \rightarrow \neg p$
T	F	F	F	F	T	F

2. Determine *all* T and F values for the logic propositions p, q, r, s, and t such that the following are F:
    a. $((p \wedge q) \wedge r) \rightarrow (s \vee t)$

       The left hand side must be T, hence, p = q = r = T. The right hand side must be F, hence s = t = F.

    b. $((p \wedge q) \wedge r) \rightarrow (s \oplus t)$

       The left hand side must be T, hence, p = q = r = T. The right hand side must be F, hence there are two cases with s = t: either both T *or* both F.

3. Using the laws in Table X (p. x in the textbook and pp. 14-15 in the class notes), simplify the logic proposition $(p \vee q) \wedge \neg(\neg p \wedge q)$ as much as possible and identify which laws you use per step.

$(p \vee q) \wedge \neg(\neg p \wedge q)$	$\leftrightarrow (p \vee q) \wedge (\neg\neg p \vee \neg q)$	DeMorgan's law
	$\leftrightarrow (p \vee q) \wedge (p \vee \neg q)$	Double negation law
	$\leftrightarrow p \vee (q \wedge \neg q)$	Distributive laws
	$\leftrightarrow p \vee F$	Inverse law
	$\leftrightarrow p$	Identity

4. Consider each of the following arguments. If the argument is valid, idenitify the rule of inference that establishes its validity. If not, indicate whether the error is due to an attempt to argue by the converse or inverse.
    a. Joe can program in C++ and also in Fortran. Therefore Joe can program in C++. *Valid by Rule of Conjuctives simplification*.
    b. A sufficient condition for Jan to win the golf tournament is that Jorge not sink a birdie on the last hole. Jan won the tournament. Therefore, Jorge did not sink a birdie on the last hole. *Invalid: attempt to argue by converse*.
    c. If Meg's computer program is correct, then she will be able to complete her assignment in at most one hour. It takes Meg more than one hour to complete the assignment. Therefore, Meg's computer program is incorrect. *Valid by Rule of Modus tollens*.
    d. Bill's car keys are either locked in the car or on the kitchen counter. Bill's car keys are not on the kitchen counter. Therefore, Bill's keys are locked in the car. *Valid by Rule of Disjunctive syllogism*.
    e. If interest rates fall, then the stock market will rise. Interest rates are not falling. Therefore, the stock market will not rise. *Invalid: attempt to argue by inverse*.

5. $a_1 = 5$ and $a_2 = 4$. $m_1 = 7$ and $m_2 = 11$. $m = 11\times7 = 77$. $M_1 = 11$ and $M_2 = 7$. We have to calculate inverses:

   $$y_1 = 11^{-1} \bmod 7 = 2 \text{ and } y_2 = 7^{-1} \bmod 11 = 8.$$

   So,

   $$x = a_1M_1y_1 + a_2M_2y_2 \bmod m = (5\times11\times2 + 4\times7\times8) \bmod 77 = 334 \bmod 77 = 26 \bmod 77.$$

6. (a) $22 = 2\times11$, 71 is prime, $105 = 3\times5\times7$

   (b) (i) 34 mod 4 = 2 and 5 mod 4 = 1. Unequal, so false.

   (ii) 34 mod 2 = 0 and 4 mod 2 = 0. Equal, so true.

   (c) The theorem in the book and class notes cannot be applied directly. There is really no satisfactory answer.

7. The first set of nested loops takes 15(n-1), which O(n). The second set of nested loops takes n(n/2), which is $O(n^2)$. The addition theorem applies and $O(n^2)$ subsumes O(n). Hence, the algorithm is $O(n^2)$.