Name: Weighted Score: 0-15 16-30 31-35 36-40

CS 275 Sections 001 and 002 Final Exam May 3, 2007

You may use the class notes from the course web site, your own notes, and the textbook only for references during the exam. You have approximately 120 minutes.

Write your name and the page number on each piece of paper you use. If you use the back, label that, too. If you split a problem's solution, please note this fact. Show your work. *There are 10 problems on two sides/page*. The problems are worth 15, 12, 6, 6, 8, 10, 10, 15, 10, and 8 for a total of 100 points. *Circle all problem numbers you answer*.

No electronic devices are to be used. Turn off your cell phone now. Answering one means you are done with your exam on the spot.

1. 15 pts: Let f: $B^4 \to B$ be a Boolean function of degree 4 defined by

$$f(a,b,c,d) = a \cdot (b + c + d) + b \cdot (a + c + (d \cdot a)) + c \cdot (\overline{a} + \overline{b} + a \cdot b \cdot c \cdot d)$$

   (a) 2 pts: When is f positive?
   (b) 2 pts: When is f negative?
   (c) 4 pts: What method would you use to simplify f if you only had 1 minute to do so? Describe the method in complete detail.
   (d) 7 pts: Write a *minimal* length code fragment (i.e., no headers, declarations, etc.) in C++ to evaluate f for all possible inputs.

2. 12 pts: Let $A_i = \{ -i, -i+1, \ldots, -1, 0, 1, \ldots, i-1, i \}$.
   (a) 2 pts: Define A_i recursively.
   (b) 2 pts: What is $\bigcup_{i=1}^{n} A_i$?
   (c) 2 pts: What is $\bigcap_{i=1}^{n} A_i$?
   (d) 3 pts: Use mathematical induction to prove your solution to (a).
   (e) 3 pts: Use mathematical induction to prove your solution to (b).

3. 6 pts: Find
   (a) 2 pts: lcm(104,22)
   (b) 2 pts: gcd(104,22)
   (c) 2 pts: gcd(104,22) $\times$ lcm(104,22) = what?

4. 6 pts: Use Big-O notation (and provide details on your answers) to describe
   (a) 2 pts: The number of elements in the power set of a finite set A.
   (b) 2 pts: The number of elements in A_i in problem 2.
   (c) 2 pts: The running time to compute f_n, the n^{th} Fibonacci number.

5. 8 pts: For $(x+y)^{50}$,
   (a) 4 pts: How many terms are there after collecting like terms?
   (b) 4 pts: What is the coefficient of x^5y^{45}? (You do not need to calculate it explicitly.)

6. 10 pts: Prove the following: $1=\binom{n}{0}<\binom{n}{1}<\binom{n}{2}<\ldots<\binom{n}{\lceil n/2\rceil}>\ldots>\binom{n}{n-1}>\binom{n}{n}=1$ for $n\in\mathbf{N}$.

7. 10 pts: Consider a standard 52 card playing deck and 5 card poker. In the questions below, you do not have to produce a number, just use the correct notation. Assume only 5 cards have been dealt to you.

(a) 5 pts: How many combinations of card hands are there?

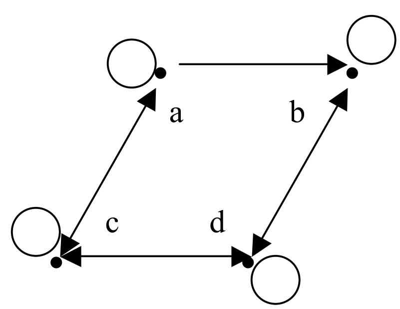
(b) 5 pts: What is the probability of having three cards of one kind?

8. 15 pts: What is the solution to the recurrence relation $a_n = 3a_{n-1} + 2a_{n-2}$ for $n > 1$ when $a_0 = 1$ and $a_1 = 2$?

9. 10 pts: Consider the four vertex graph representation of a relation below, where the circles are loops.

(a) 6 pts: Determine the properties of the relation (reflexive, symmetric, antisymmetric, or transitive).

(b) 4 pts: What needs to be added to the graph (if anything) to make this an equivalence relation.



10. 8 pts: Use on an alphabet of 6 letters with given frequencies: (A,1.2), (B,0.5), (C,0.6), (D,1.0), (E,1.2), (F,0.2).

(a) 6 pts: Construct the Huffman coding. Show all steps using trees and forests.

(b) 2 pts: What is the average number of bits used to encode a letter?