JONATHAN HU, MEI HUANG, MIKE SUKOP, DANNY
THORNE, KAREN WALTERS

FOOTBALL LIGHTING OPTIMIZATION

MATHEMATICS 721

SELECTED TOPICS IN NUMERICAL ANALYSIS:

MATHEMATICAL MODELS OF PHYSICAL PROBLEMS

FINAL PROJECT REPORT

PROFESSOR CRAIG C. DOUGLAS

FOOTBALL LIGHTING OPTIMAZATION

MATHEMATICS 721:

SELECTED TOPICS IN NUMERICAL ANALYSIS:

MATHEMATICAL MODELS OF PHYSICAL PROBLEMS

FINAL PROJECT REPORT

PROFESSOR CRAIG C. DOUGLAS

INTRODUCTION

This group-effort class project has been completed as part of course requirements for MA 721 (Selected Topics in Numerical Analysis: Mathematical Models of Physical Problems). Professor Douglas gave a general description of the problem. We collected background information, developed a conceptual model, and constructed a numerical model to address the problem.

All sports that are played under floodlights have a set of criteria for the illumination of the sports ground. First, the level of illumination must be as high as possible. Second, it must be as uniform as possible. Depending on the level of competition, the criteria are different. There are numerous types of floodlights available to meet the desired illumination. For each type of floodlight, tables of photometric data that specify the output in different directions are available from the manufacturers. However, with 36, 144, or 240 floodlights on masts around a field, it becomes very difficult to know where to aim each of the floodlights.

Our goal in this project was to develop a method to tell the groundskeepers where each floodlight should be aimed to get the required illumination on the playing field.

DOCUMENT ORGANIZATION

This report is organized as follows:

- First, sports lighting basics are considered. This includes sections on terminology, lighting geometry, photometric data, aiming diagrams, and a historical perspective of the sports lighting optimization problem. This section is primarily the result of background research conducted by the project team.

- The second major portion of the document discusses the conceptual illumination uniformity optimization model we developed to address the sports lighting issue.

- Finally, the results of modeling efforts, including the code we developed and some output are presented in the last major part of the report. A brief discussion on modeling difficulties is included.

SPORTS LIGHTING BASICS

There are a number of issues that are critical to a successful sports lighting installation. First, to ensure adequate intensity for various purposes (e.g., televised vs. non-televised events), it is essential that the intensity of the illumination be quantifiable. This requires that a number of standard measures of luminous intensity be defined. There are also a number of lighting industry standard terms that should be adopted in work of this nature to make it more meaningful to those in the industry and to enhance communication between professionals with different backgrounds. Thus, we begin with a section devoted to terminology.

There are also fundamental geometric criteria related to glare, light utilization, illumination uniformity (in three as well as in two dimensions), shadows, and stadium size. These can easily make the difference between a successful lighting project and spectator annoyance, resource waste, and even significant impacts on the players' ability to compete in the sport. Some of these criteria are discussed under the section entitled 'Lighting Geometry'.

Photometric data, which quantify the output of a lighting fixture in terms of angles from its primary focal point, are discussed in their own section.

Aiming diagrams are reviewed in the next section. They are used to transfer optimal light orientation information to sports groundskeepers. Construction of aiming diagrams was the primary goal of our study.

Finally, a brief historical excerpt summarizes the standard approach to lighting optimization in 1969. It is presented under 'Historical Perspective on the Point Method'.

TERMINOLOGY

The following definitions were compiled from two Internet sources.

CANDELA, CD: the SI unit of luminous intensity. One candela is one lumen per steradian. Formerly, candle.

LUMINAIRE: A general term for a complete lighting unit. It includes the housing, the reflector, lens and lamps. (Colloquial terms include light, lantern, fixture, unit, instrument, fitting.)

LUMEN: An amount of light energy within an area. The lumen is the unit of 'luminous flux' and is defined as the amount of light which falls on one square meter of a surface at a constant distance of one metre from a source of one candela.

LUMEN,LM: SI unit of luminous flux. Radiometrically, it is determined from the radiant power. Photometrically, it is the luminous flux emitted within a unit solid angle (one steradian) by a point source having a uniform luminous intensity of one candela.

LUX,LX: the SI unit of illuminance. One lux is one lumen per square meter (lm/m2).

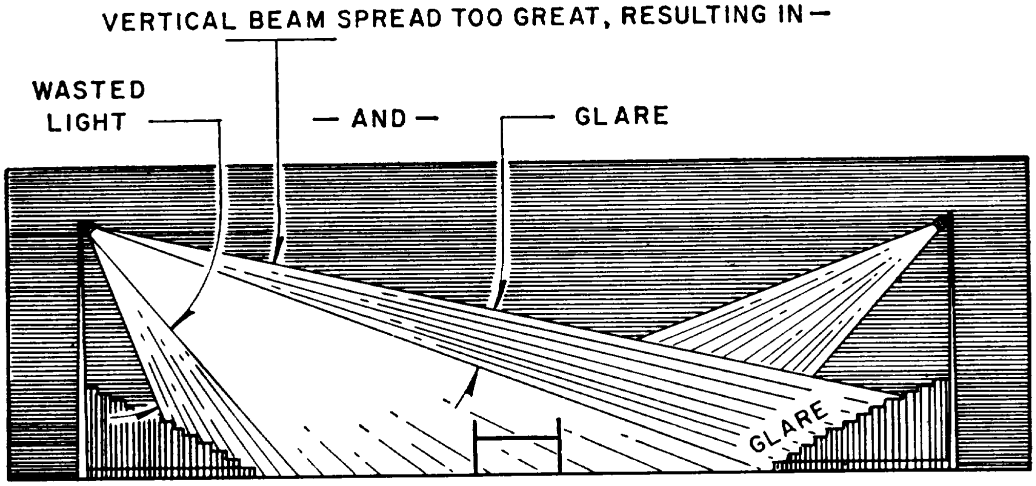
NADIR: when used in lighting, the point directly below the center of the luminaire.

POINT METHOD: a lighting design procedure for predetermining the illuminance at various locations in lighting installations, by use of luminaire photometric data.

FOOTCANDLE, FC: the unit of illuminance when the foot is taken as the unit of length. It is the illuminance on a surface one square foot in area on which there is uniformly distributed flux of one lumen, or the illuminance produced on a surface all points of which are at a distance of one foot from a directionally uniform point source of one candela.

LIGHTING GEOMETRY

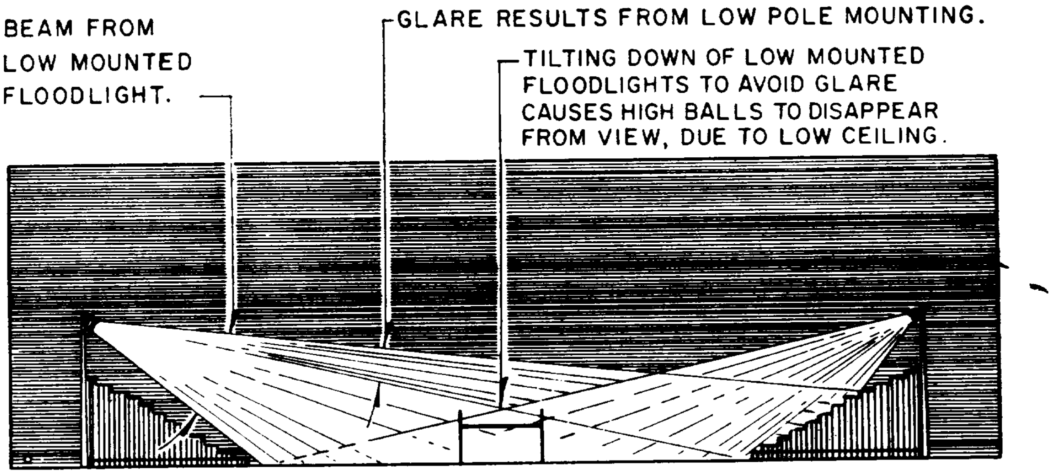
Glare can make viewing sporting events uncomfortable for spectators and can cause significant problems for players. Light that falls off of the playing field (e.g., on the backs of spectators) serves little purpose and is wasteful. Figure 1 illustrates how these problems can arise when the vertical spread of the light beam is too large.



Floodlights with too great a vertical beam spread waste light and cause glare.

Figure 1

Mounting height can also play an important role in causing glare for spectators. If the beams are tilted down to avoid glare, high flying balls will tend to disappear from view, causing difficulty for both players and spectators. Figure 2 illustrates these problems.



Sports floodlights mounted on poles that are too low either cause glare in the spectators' eyes or do not illuminate high-flying balls.

Figure 2

Shadows are also an issue in basic lighting geometry. In practice, the number of shadows produced characterizes the quality of lighting. More shadows produced by more illumination sources are generally preferable to harsh shadows produced by a small number of sources. Figure 3 illustrates the reduction in shadow intensity as the number of light sources increases.

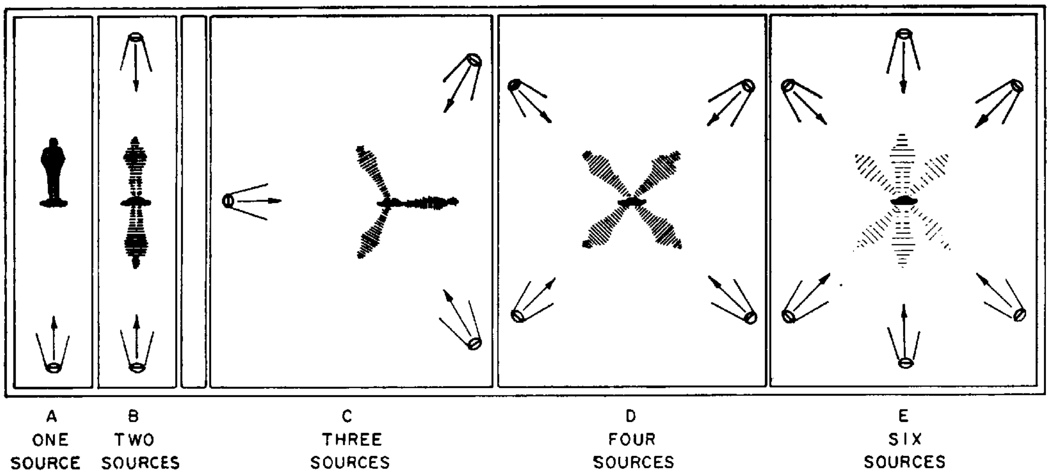


Figure 3

The efficiency of light utilization is also an issue. Figure 4 shows a comparison between eight National Electrical Manufacturers' Association (NEMA) Type 4 beam spread (46 - 70°) luminaires and six NEMA Type 3 luminaires (29 - 46°). The six luminaires with smaller beam spread have an appreciably higher level of light utilization, similar illuminace levels, and significantly lower cost.

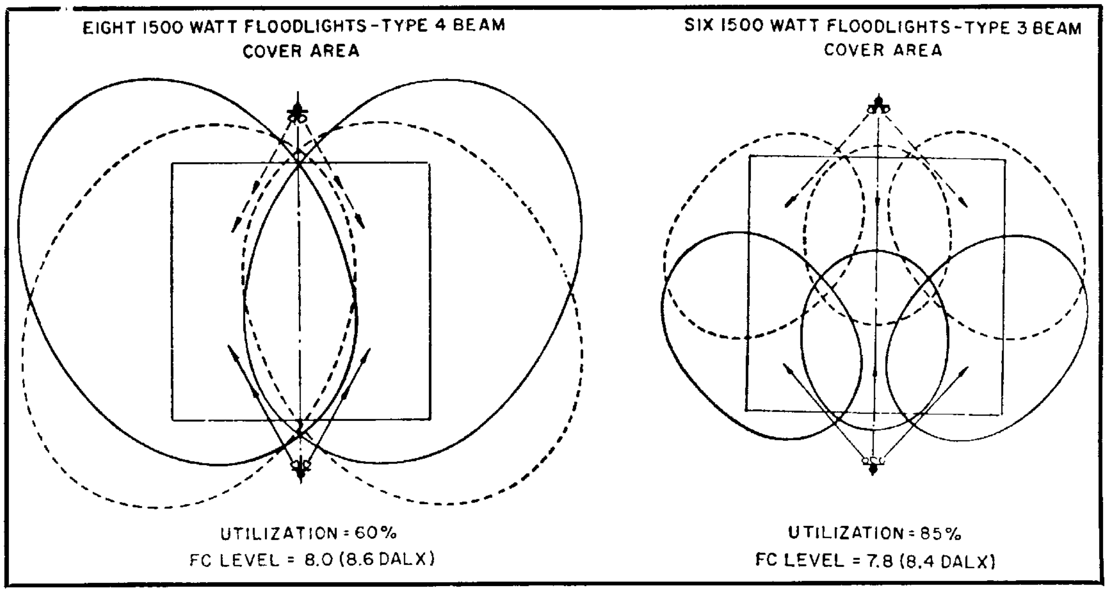
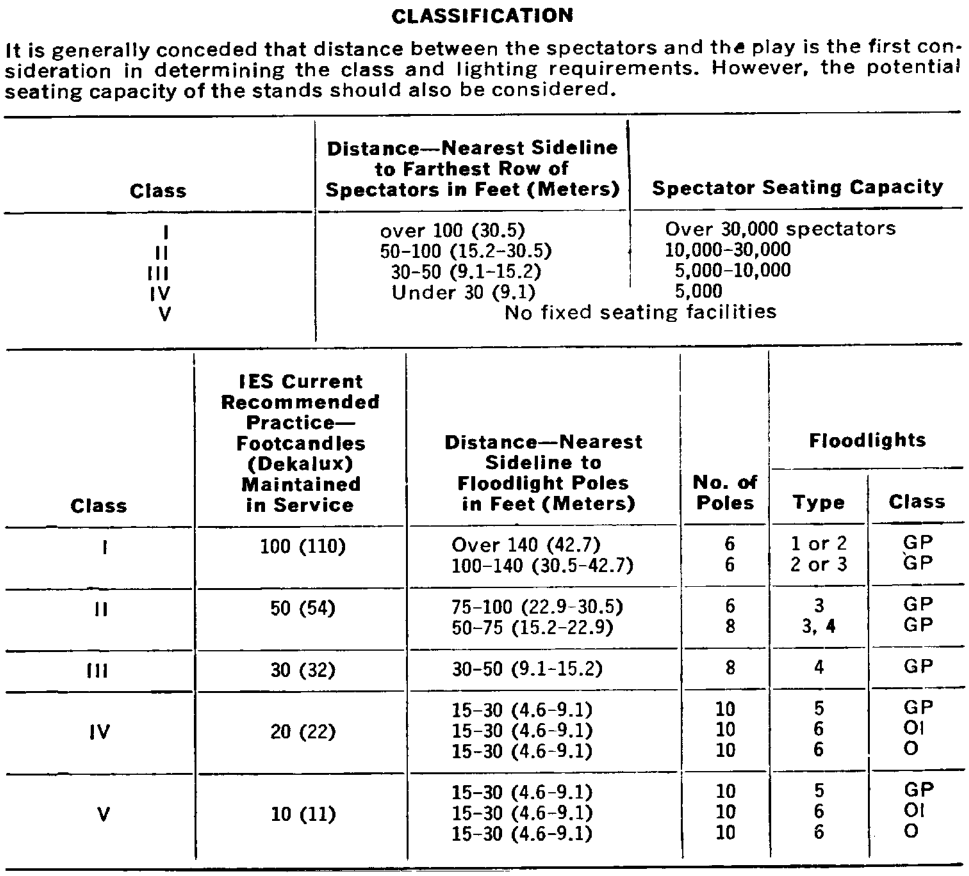


Figure 4

The Illuminating Engineering Society (IES, 1969) has divided regulation football stadiums into five classes depending primarily on the distance from the sidelines to the most distant spectator. The stadium's seating capacity is also considered. Table 1 contains the classification scheme, the recommended illuminance, the distance to the luminaire masts, the number of poles, and the type

(NEMA 1, 2, 3, 4, 5, or 6) of floodlight.



CLASSIFICATION

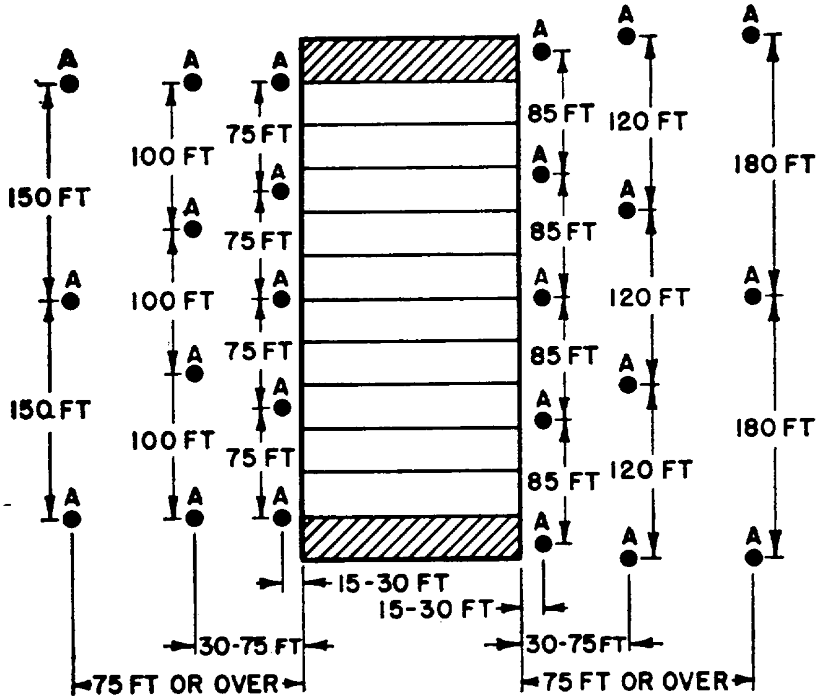
It is generally conceded that distance between the spectators and the play is the first consideration in determining the class and lighting requirements. However, the potential seating capacity of the stands should also be considered.

Class	Distance—Nearest Sideline to Farthest Row of Spectators in Feet (Meters)	Spectator Seating Capacity
I	over 100 (30.5)	Over 30,000 spectators
II	50-100 (15.2-30.5)	10,000-30,000
III	30-50 (9.1-15.2)	5,000-10,000
IV	Under 30 (9.1)	5,000
V	No fixed seating facilities	

Class	IES Current Recommended Practice—Footcandles (Dekalux) Maintained in Service	Distance—Nearest Sideline to Floodlight Poles in Feet (Meters)	No. of Poles	Floodlights	
				Type	Class
I	100 (110)	Over 140 (42.7)	6	1 or 2	GP
		100-140 (30.5-42.7)	6	2 or 3	GP
II	50 (54)	75-100 (22.9-30.5)	6	3	GP
		50-75 (15.2-22.9)	8	3, 4	GP
III	30 (32)	30-50 (9.1-15.2)	8	4	GP
IV	20 (22)	15-30 (4.6-9.1)	10	5	GP
		15-30 (4.6-9.1)	10	6	OI
		15-30 (4.6-9.1)	10	6	O
V	10 (11)	15-30 (4.6-9.1)	10	5	GP
		15-30 (4.6-9.1)	10	6	OI
		15-30 (4.6-9.1)	10	6	O

Table 1

Figure 5 shows a plan view of pole positions for the combinations of different numbers of floodlight poles and pole positions.



Any of the above six pole plans or any intermediate longitudinal spacings are considered good practice with local field conditions dictating exact pole locations.

Figure 5

Football inolves a combination of aerial and ground play. According to the IES, it requires 'adequate' lighting to 50 feet above ground. Adequate uniformity is attained when "…the ratio of maximum to minimum illumination does not exceed 3 to 1…" (IES, 1961). A principal reason for this is that flying balls appear to accelerate on passing from light to dark space. This can cause the player's judgement of trajectories to be distorted.

PHOTOMETRIC DATA

Photometric data for a luminaire are typically provided by the manufacturer. The format of the data is usually tabular and the data consist of luminaire output in lumens for different vertical and horizontal angles from the main axis of the luminaire. The luminaire output can be radially symmetric or can be more complex. A wide varitey of output options are available for various purposes. In conjunction with this project, we obtained photometric data for the luminaires used at the University of Kentucky's Commonwealth Stadium. They are attached to this report as Appendix A.

In addition, we obtained photometric data for General Electric's Ultrasport floodlight. Data for that luminaire are shown in Figure 6.

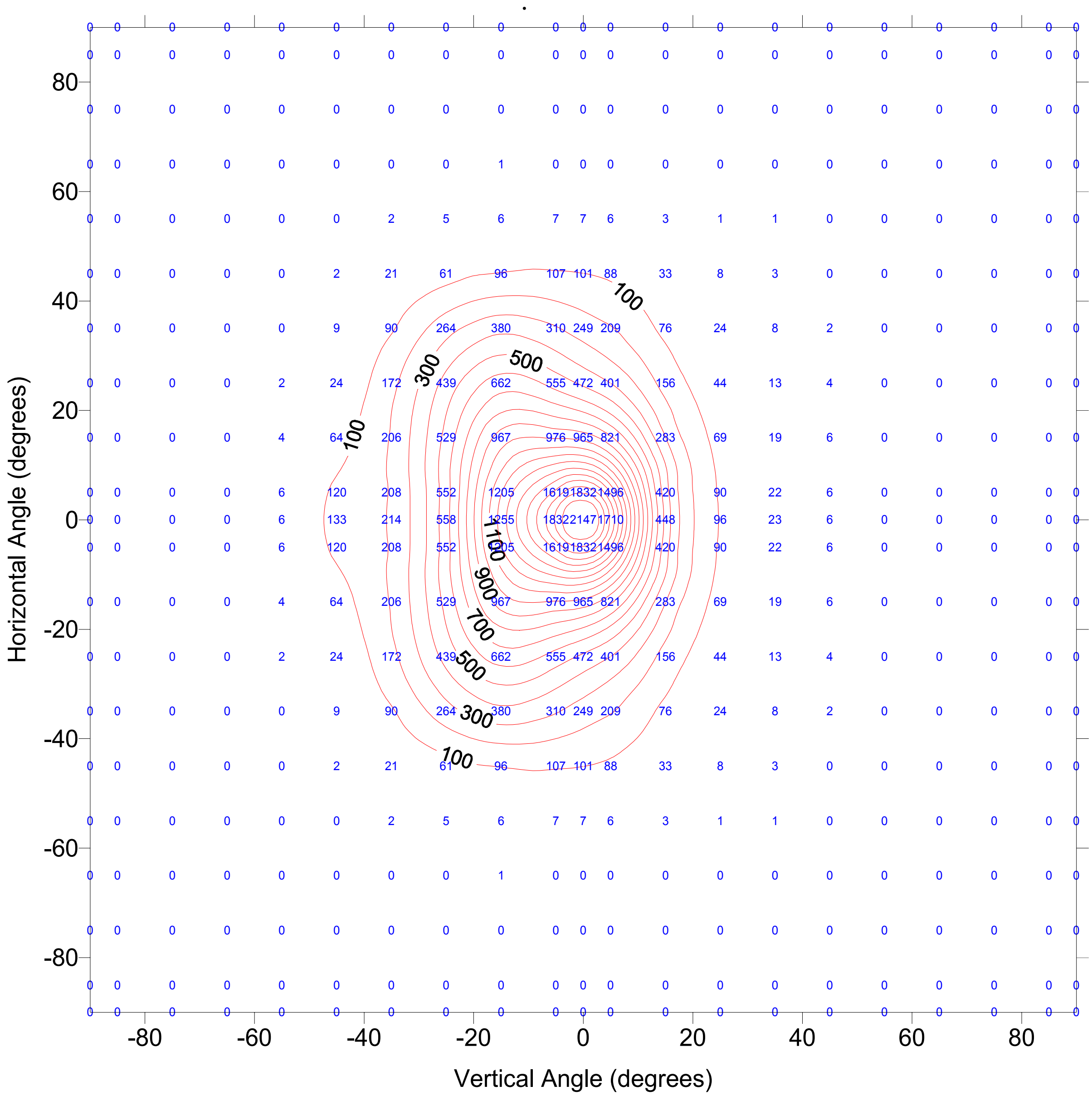
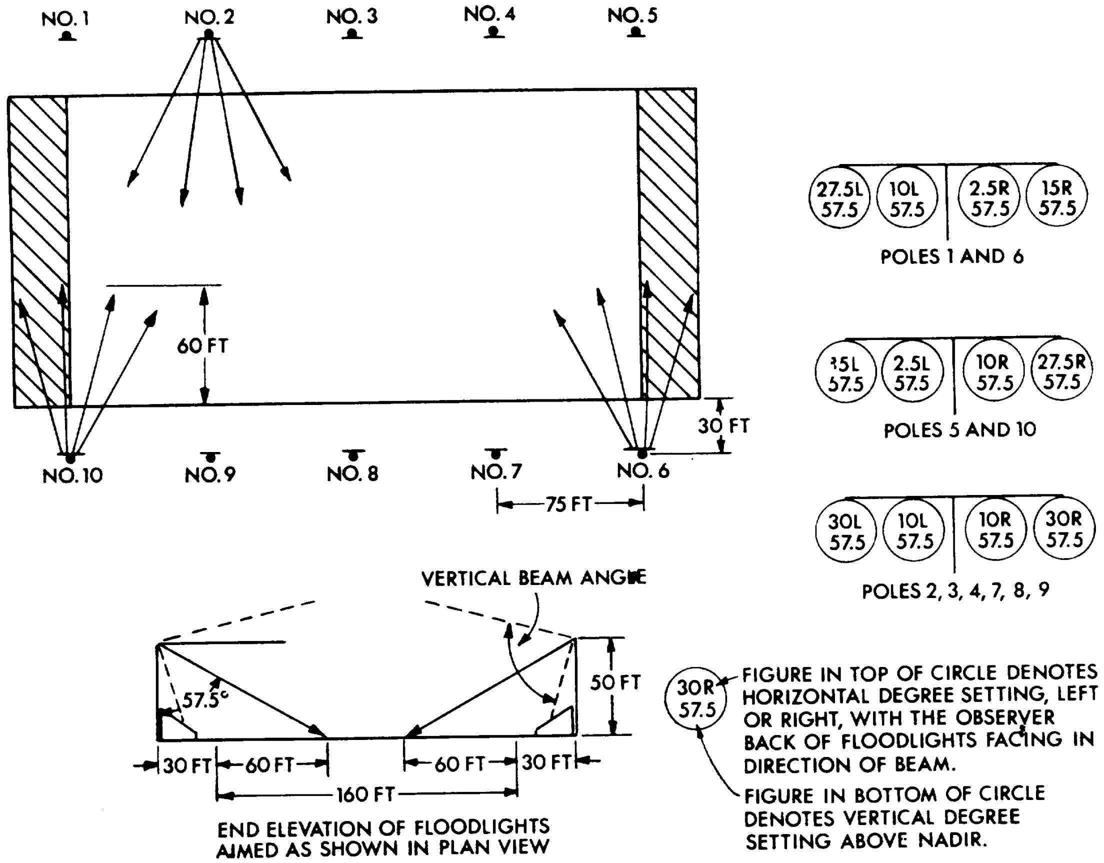


Figure 6
Clearly, this luminaire has a strongly asymmetric output pattern, where the drop off with increasing vertical angle (measured from the aiming angle) is much steeper than the drop off with horizontal angle. The luminaires used in most football stadiums probably have much more symmetric output. (See Appendix A).

AIMING DIAGRAMS

Aiming diagrams are typically supplied with sports lighting installations of appreciable size. We obtained the aiming diagram for Commonwealth Stadium as part of the project research. It is included as Appendix B. An aiming diagram for a smaller installation is provided in Figure 7.



: 9.1 m; 35 ft
60 ft = 18.3

(b) Angular aiming diagram for 40-unit football field.

Figure 7

At least two types of aiming diagrams can be specified. In a ‘manual’ aiming diagram, only the point to which the beam should point on the field is specified – usually in a map view. This is the type of diagram provided to the University of Kentucky (Appendix B). The second type of aiming diagram is known as an ‘angular’ aiming diagram because the aiming angles are provided explicitly. Details are provided in Figure 7.

Construction of aiming diagrams to provide optimal uniformity was the ultimate goal of this project. The computer program developed for the project (Appendix C) accepts and provides map (‘manual’) and angular data with similar ease.

Figure 8 shows workers laying out aiming points in the field in preparation for manual aiming.

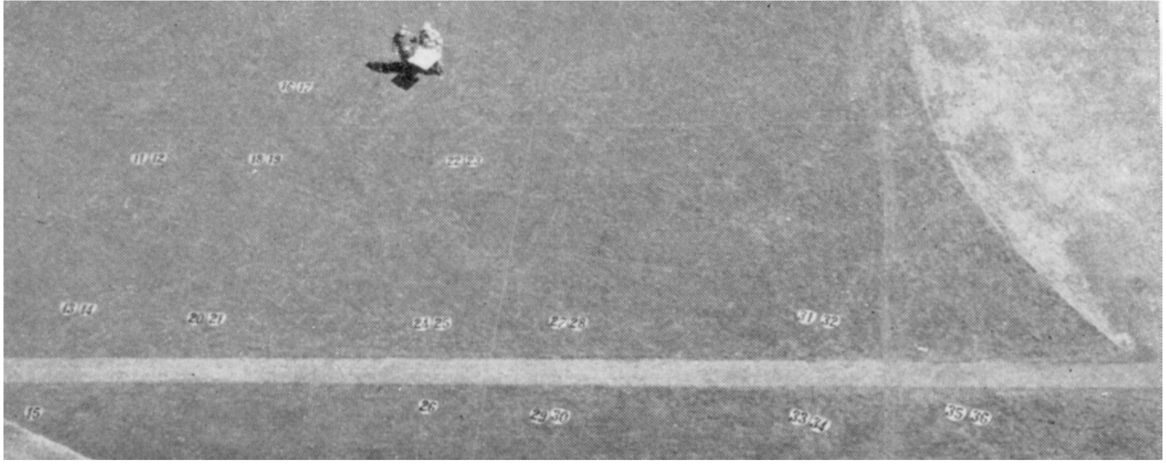


Figure 8

Figure 9 shows the view from the height of the luminaires, where workers responsible for aiming must physically adjust luminaire angles

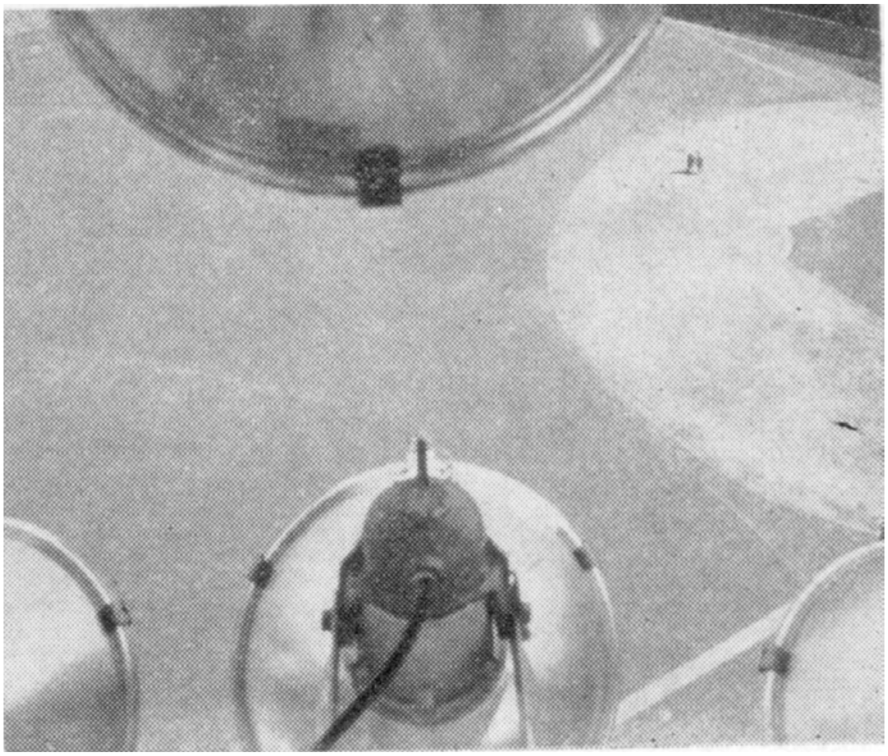


Figure 9

HISTORICAL PERSPECTIVE ON THE POINT METHOD

As noted in the terminology section, the 'point method' is a procedure for predetermining the illuminance at various locations in lighting installations, by use of luminaire photometric data. This is the approach adopted here.

In the late 1960's, computational power was just beginning to become available. The Illuminating Engineering Society wrote the following about the point method in 1969:

> "Calculation methods make it possible to pre-determine the foot candle (lux) distribution provided by any given aiming pattern. However, because such calculations are long and tedious, it is general practice to base spotting or aiming diagrams … on scale plots of the beam spread and the area to be lighted, previous calculations, and practical experience…"

The principal motivation for this project was the ready availability of enormous computing power that can be brought to bear on the sports lighting problem today. It should be possible to explore this problem in detail and possibly obtain solutions that provide more uniform illuminance than previously possible.

MATHEMATICAL APPROACH

The general objective here is to arrange the lights (in terms of their horizontal and vertical angles, theta and phi, respectively) in order that the level of illuminance is equal at each point in the field. The next diagram (Figure 10) shows how phi and theta are measured. The angles alpha and beta are discussed below.

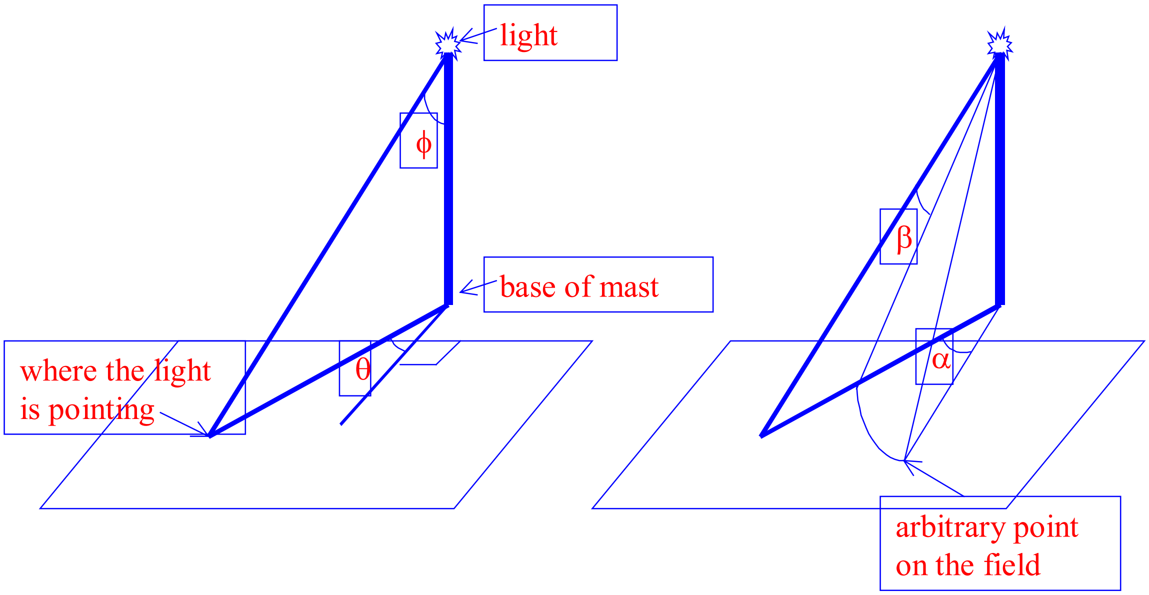


Figure 10

We approach this task by considering a discrete subset of the points on the field and comparing the illuminance between every pair of those points. The differences in illuminance between these pairs of points on the field are taken as a system of functions to be minimized in some way. In particular, we wish to minimize the sum of the squares of these functions with respect to the orientation of the lights. This can be illustrated as follows:

$$\underset{\substack{\theta_s,\phi_s \\ 1\le s\le NumLights}}{Min} \sum_{(i,j)\neq(k,l)} \left(\sum_{t=1}^{NumLights} I_{ij}(\theta_t,\phi_t) - \sum_{t=1}^{NumLights} I_{kl}(\theta_t,\phi_t) \right)^2$$

$$\text{where } I_{ij}(\theta,\varphi) = \frac{I_0(\alpha_{ij}(\theta),\beta_{ij}(\phi))}{D_{ij}^{\;2}} \text{ and } D_{ij} = \sqrt{(x_{ij}-x_0)^2+(y_{ij}-y_0)^2+z_0^{\;2}}$$

where the mast is at (x0,y0,z0).

The tricky part of this is computing the initial illuminance coming from a given light toward a given point on the field. The vendor provides a table of these values based on the horizontal and vertical angle from the center of the light, which we've denoted by alpha and beta. However, we need to be able to reference a table of these values based on the orientation of the light and the coordinates of the point on the field. In other words, we need for alpha and beta to be functions of theta and phi and the coordinates of points on the field. These functions look like this:

Let $q = \dfrac{y_{ij}-y_0}{x_{ij}-x_0}$.

If $q > 0$ then $\alpha_{ij}(\theta) = \theta - \tan^{-1}(q)$.

If $q < 0$ then $\alpha_{ij}(\theta) = 180 - \tan^{-1}(q) - \theta$.

$$\beta_{ij}(\phi) = \tan^{-1}\left(\frac{\sqrt{(x_{ij}-x_0)^2+(y_{ij}-y_0)^2}}{z_0}\right) - \phi.$$

Hence, we can conveniently construct the aforementioned system of functions by traversing the field and doing the computations based directly on the orientation of the lights and the coordinates of our current location on the field. This process is outlined in more detail in the next section.

RESULTS

We chose to implement the sports lighting model in Fortran 77 and C. The illuminance software has several major parts:

- Initialization
- Initial illuminance calculation
- Function evaluation and Jacobian evaluation
- Nonlinear least squares optimization (a MINPACK routine from http://netlib.org)

The initial illuminance calculation is based on data stored in a photometric table. As mentioned before, for a given grid point, there is an associated vertical and horizontal angular displacement from the floodlight's angular aiming coordinates. The table is arranged so that columns represent the vertical angular displacement and rows represent the horizontal angular displacement. The photometric data is interpolated for any grid point which does not appear in the table.

For each grid point, the total illumination (due to all lights) is calculated for the current aiming pattern. Thus, the functions to be minimized are evaluated in the following manner:

```
for I = 1 to NumberofGridPoints
    for J = I to NumberofGridPoints
        Calculate F(I,j) = difference in illumination between grid points I and J
    end for
end for
```

The Jacobian evaluation is performed in a similar manner. Derivatives are approximated by central differences.

The heart of our illuminance code is a MINPACK routine written by Garbow, Hillstrom, and More. This routine implements a variant of the Levenberg-Marquardt algorithm. The MINPACK routine makes repeated calls to the subroutines which make the function evaluations and Jacobian evaluations.

Finally, we used the Matlab 5.0 package for data visualization. A complete listing of the code is in Appendix C.

TESTING AND RESULTS

As part of the testing of the computer code, two simple, single luminaire initial conditions were considered.

Figure 11 shows the first test case in which a 10 yard high luminaire at coordinates (0,-25) points towards the center of the field. This test case demonstrates the effect of the r^2 drop off in illumination intensity with distance from the light source (i.e., the intensity is highest closer to the source even though the luminaire is aimed at the center of the field).

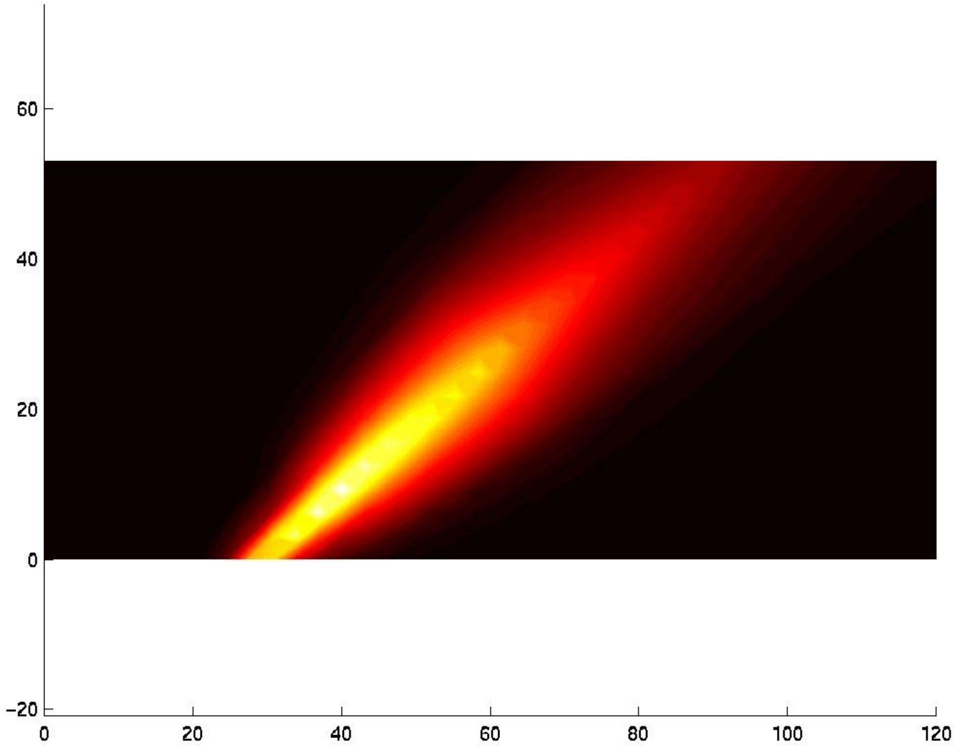


Figure 11

Figure 12 illustrates the same general conditions except that the mast height was increased to 100 yards. This has the effect of greatly reducing the spread of the illumination across the field. The brightest area is at the aiming point and the spread of the illumination is controlled by the photometric data and the angle of incidence on the surface.

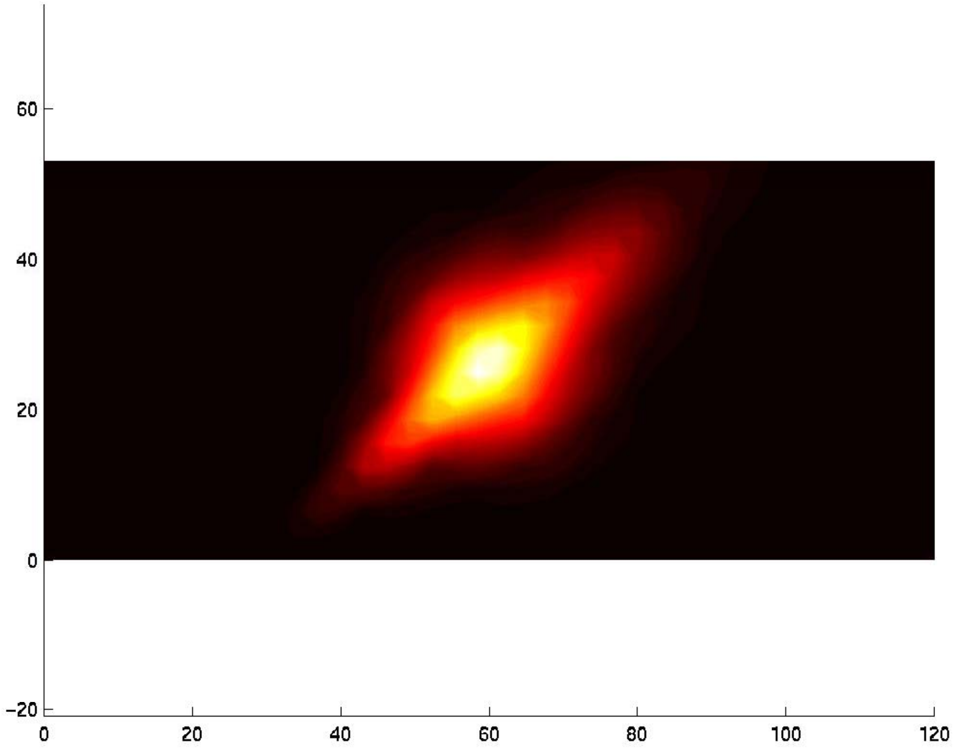
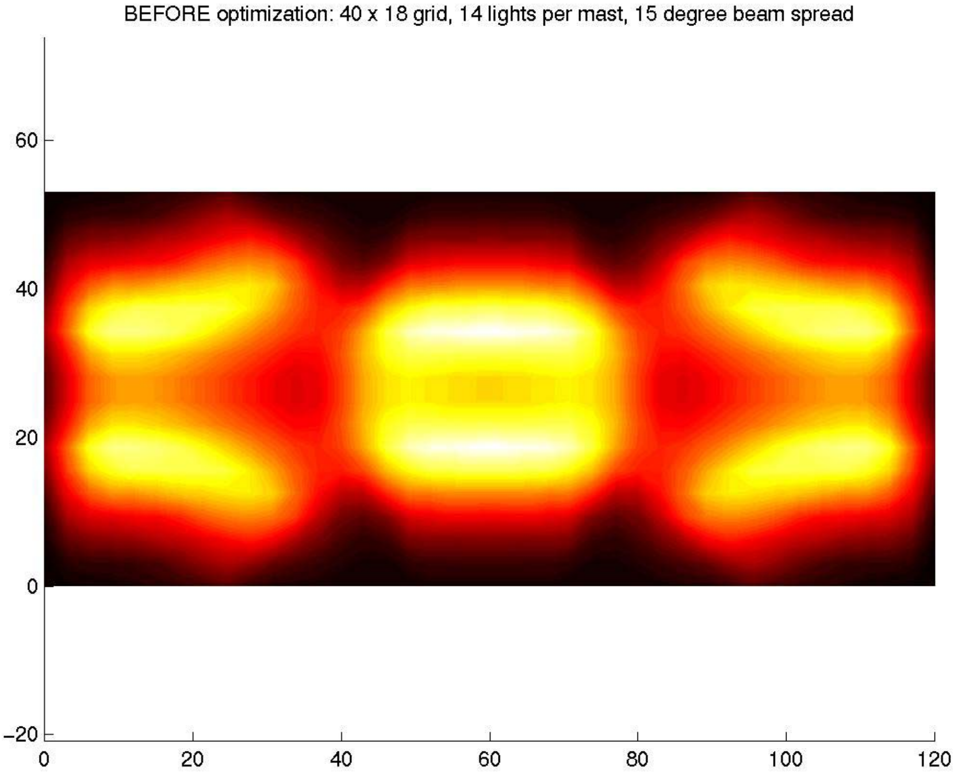


Figure 12

We assume that the field is 120 yards long and 53 yards wide. The lower left hand corner is placed at the origin. There are 6 light masts, located at the points (0,-25), (60,-25), (120,-25), (0,78), (60,78), and (120,78). We assume a mast height of 25 yards and 14 lights per mast. We make no distinction between lights on a given mast, i.e., all lights on a mast are assumed to have the same spatial coordinates. We use a photometric table for a luminaire with a 15-degree beam spread. All stopping tolerances were assumed to be 1.0e-6. Below is a contour plot of an initial guess for a 40 by 18 grid.



Below (Figue 13) is the solution from our program.

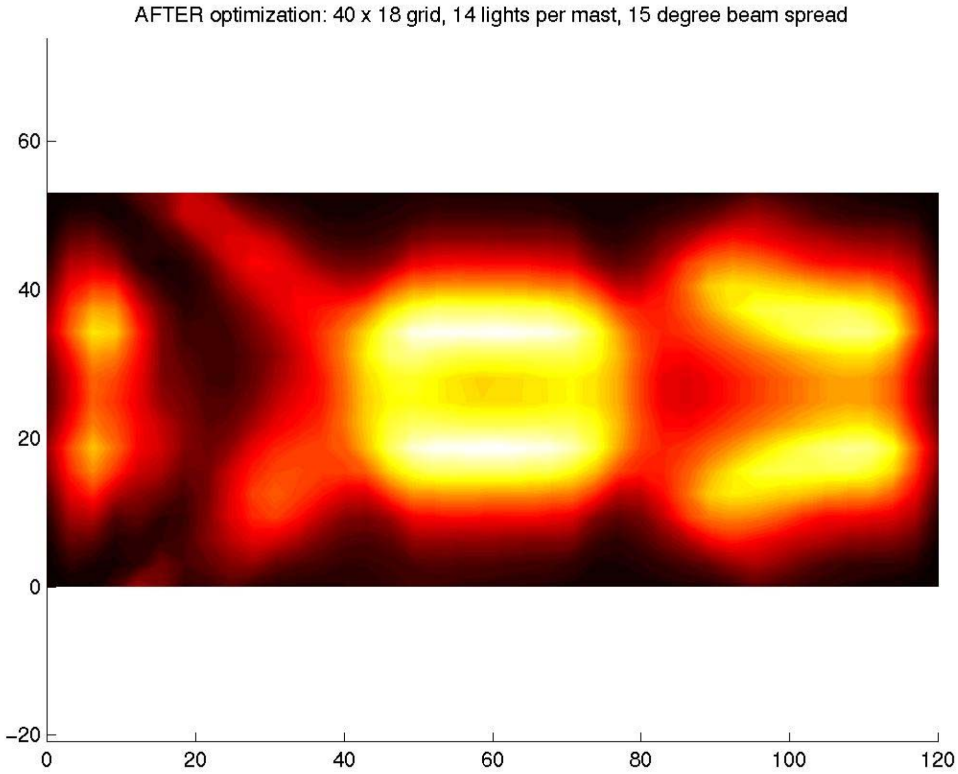

AFTER optimization: 40 x 18 grid, 14 lights per mast, 15 degree beam spread
60
40
20
0
−20
0
20
40
60
80
100
120

Figure 13

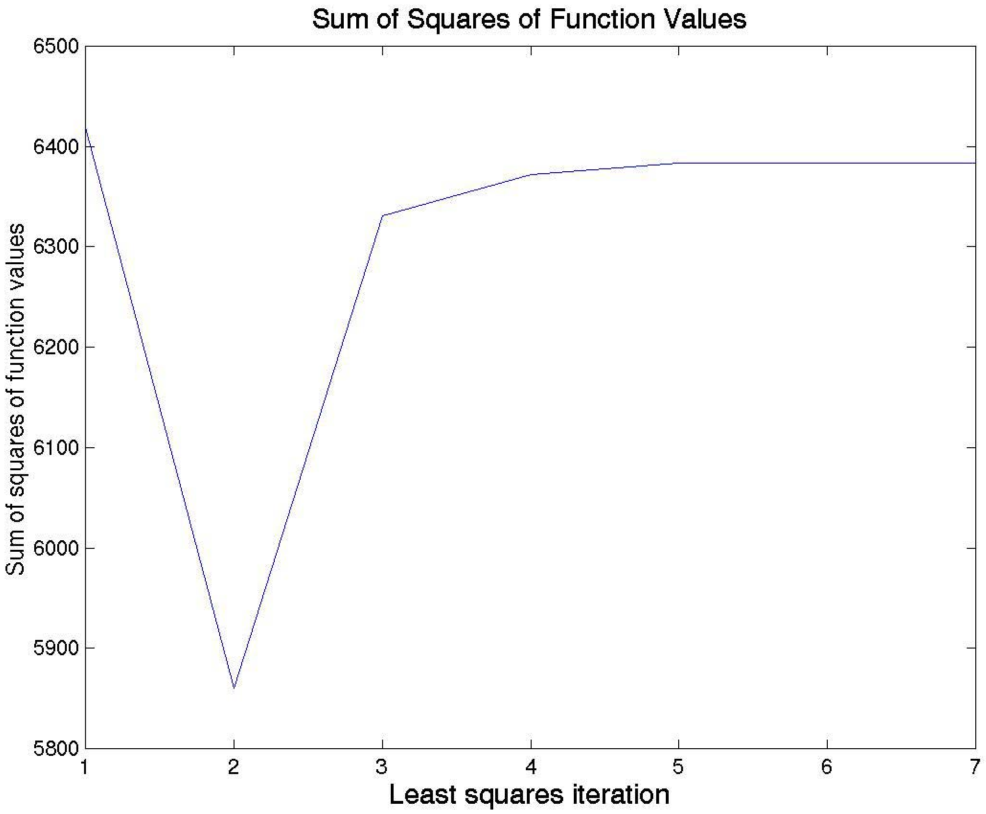


Figure 14

Clearly, the output is not a good solution to the illuminance problem. Based on our plots of initial guesses, we believe that our function evaluation is correct. The Jacobian evaluation is a simple extension of this code. This leaves the MINPACK software as a possible problem source, although the least squares subroutine does successfully minimize simple test functions.

Another possible problem is our choice of initial solution. If the problem is very sensitive to choice of initial guess, then our initial guesses may simply be too "far" from a desired solution.

One avenue which we have not explored is using the data from Commonwealth Stadium on the University of Kentucky campus as an initial guess. We assume that the current aiming scheme is acceptable and so should generate an even better solution, if our program works correctly.

Figure 15

APPENDIX A. PHOTOMETRIC DATA FOR COMMONWEALTH STADIUM LUMINAIRES

APPENDIX B. AIMING DIAGRAM FOR COMMONWEALTH STADIUM

APPENDIX C. PROJECT COMPUTER CODE