

Environmental Modeling of the Spread of Road Dust

Craig C. Douglas

University of Kentucky (CS/CCS/ME)

Yale University (CS)

Based on an article by S.B. Hazra, T. Chaperon, R. Kroiss, J. Roy, and D. La Torre with advice from H. Hanche-Olsen.

Problem Definition

In places where roads are usually covered with ice and snow in winter, studded tires became common in the 1960's. The studs clear the roads, leaving bare pavement for the studs to eat into. A plume of dust forms over the roadway and spreads out along the side of the road. For example, in Norway (with only 4,000,000 people), over 300,000 tons of asphalt and rock dust is generated every winter, causing a serious health hazard.

We want to determine how the dust cloud spreads.

Outline of Solution Methods

Individual motion of a particle and its free fall velocity.

A turbulent diffusion model.

Discussion of numerical results.

Possible improvements to the model, given an infinite amount of time.

Individual Dust Particle Motion

Assume that we have still air and only one dust particle in motion. Let

ρ_{air} = density of the air

v = velocity of the particle

A = apparent cross section of the particle

f = friction factor of the air

k = kinematic viscosity of the air

The magnitude of the force corresponding to the interaction of the air is given by

$$F = .5 \rho_{\text{air}} v^2 A f$$

Example: spherical particle of diameter D , $A = (\pi/4) D^2$.

Let the **Reynolds number** be given by $Re = Dv/k$.

The friction factor $f = f(Re)$. A Stokes formula can be used when $Re < .5$. Theoretically,

$$(1) \quad f(Re) = 24/Re.$$

For $.5 \leq Re \leq 2 \times 10^5$, an empirical formula for $f(Re)$ is given by

$$f(Re) = .4 + 6/(1 + Re^{1/2}) + 24/Re.$$

Consider a spherical particle when (1) holds. The interaction of the air reduces to

$$F = 3\pi r_{\text{air}} Dv/k$$

Note that the magnitude of F is proportional to v .

The force vector \mathbf{F} has an opposite direction to the particle velocity \mathbf{v} , so

$$\mathbf{F} = -3\pi r_{\text{air}} D\mathbf{v}k$$

If the wind is present, there will be an additional force of the form

$$\mathbf{F}_{\text{wind}} = 3\pi r_{\text{air}} D\mathbf{v}k$$

The vector \mathbf{v}_{wind} is a function of both time and position usually. However, if we assume a constant vector \mathbf{v}_{wind} , then the motion of the particle is described in 2D by the pair of equations

$$\ddot{x} + Cx - Cv_{\text{wind}X} = 0 \quad (\text{horizontal component})$$

$$\ddot{z} + Cz - Cv_{\text{wind}Z} = g \quad (\text{vertical component})$$

When the particle is a homogeneous sphere of density r_p ,

$$C = 18 (kD^{-2}) (r_{\text{air}}/r_p), \quad \text{a constant.}$$

When the vertical wind velocity component is zero, then the particle vertical velocity ($v_z = \dot{z}$) is given by

$$v_z(t) = g/C + (z_0 - g/C) e^{-ct}, \quad g = \text{gravity.}$$

When t goes to infinity,

$$| v_t(t) | \text{ is bounded by } g/C.$$

The limit velocity is the particle velocity when there is no momentum along the z axis. This is the **free fall velocity** v_f . In our case,

$$(2) \quad v_f = (gD^2r_p) / (18kr_{\text{air}}).$$

Trajectory of One Particle

We use the following constants:

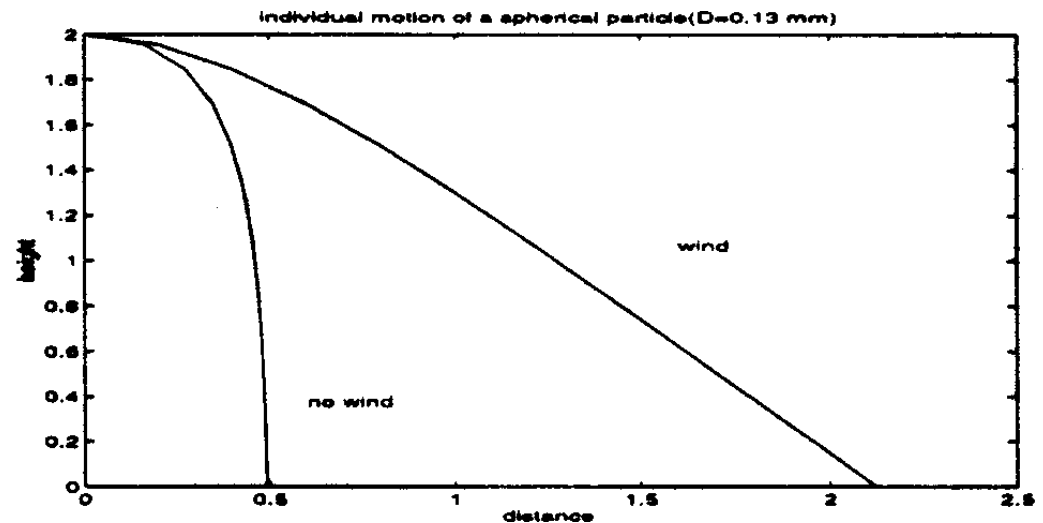
$$k = 1.5 \times 10^{-5} \text{ m}^2/\text{s}$$

$$r_{\text{air}} = 1 \text{ kg}/\text{m}^3$$

$$r_p = 4 \times 10^3 \text{ kg}/\text{m}^3$$

$$g = 9.81 \text{ m}/\text{s}^2$$

See Fig. 1 (meters).



In the case of free fall velocity, the upper bound on the particle diameter is **37 microns** to ensure that $Re < 0.5$. This is significant since particles with diameters less than **10 microns** pose a serious health hazard.

A Turbulent Diffusion Model

We include both wind velocity and the free fall tendency in a diffusion process. We assume 2D since a road provides a line source of dust and our interest is in how the dust spreads out perpendicular to the road. Let

$\phi(x,z,t)$ = concentration of dust particles.

U = constant term of the x axis wind velocity.

Consider the diffusion equation with D_x and D_z as diffusivity coefficients:

$$(3) \quad \phi_t + U\phi_x - v_f\phi_z = D_x\phi_{xx} + D_z\phi_{zz}$$

with an absorbing boundary condition

$$\phi(x,0,t) = 0$$

Other boundary conditions could have been

Reflecting: $\phi_z(x,0,t) = 0$

Mixture: $\phi_z(x,0,t) + \phi(x,0,t) = 0$

The initial conditions give us a source of particles at height h :

$$\phi(x,z,0) = \delta(x) \delta(z-h)$$

U , v_f , D_x , and D_z are normally functions of x , z , and t . To get an analytic solution, we assume that they are constants.

Now consider a dimensionless version of (3). Apply the transformation

$$t = Tt^*$$

$$x = Xx^*$$

$$z = Zz^*$$

to get

$$\phi_{t^*} (UT/X) \phi_{x^*} - (v_f T/Z) \phi_{z^*} =$$

$$(\mathcal{D}_x T/X^2) \phi_{x^*x^*} + (\mathcal{D}_z T/Z^2) \phi_{z^*z^*}$$

Forcing $UT/X = D_x T/X^2 = 1$ gives us the horizontal and time scales

$$X = D_x/U \quad \text{and} \quad T = D_x/U^2$$

Forcing $D_z T/Z^2 = 1$ gives us the vertical length scale and the nondimensional free fall velocity

$$Z = (D_x D_z)^{1/2}/U \quad \text{and} \quad v_f^* = (v_f/U) (D_x/D_z)^{1/2}.$$

Let $h^* = hU (D_x D_z)^{-1/2}$. The equations can be rewritten as

$$\phi_{t^*} + \phi_{x^*} - v_f^* \phi_{z^*} = \phi_{x^*x^*} + \phi_{z^*z^*} \quad (\text{PDE})$$

$$\phi(x^*, 0, t^*) = 0 \quad (\text{BC})$$

$$\phi(x^*, z^*, 0) = \delta(x^*) \delta(z^* - h^*) \quad (\text{IC})$$

A classical mirror image method yields the solution

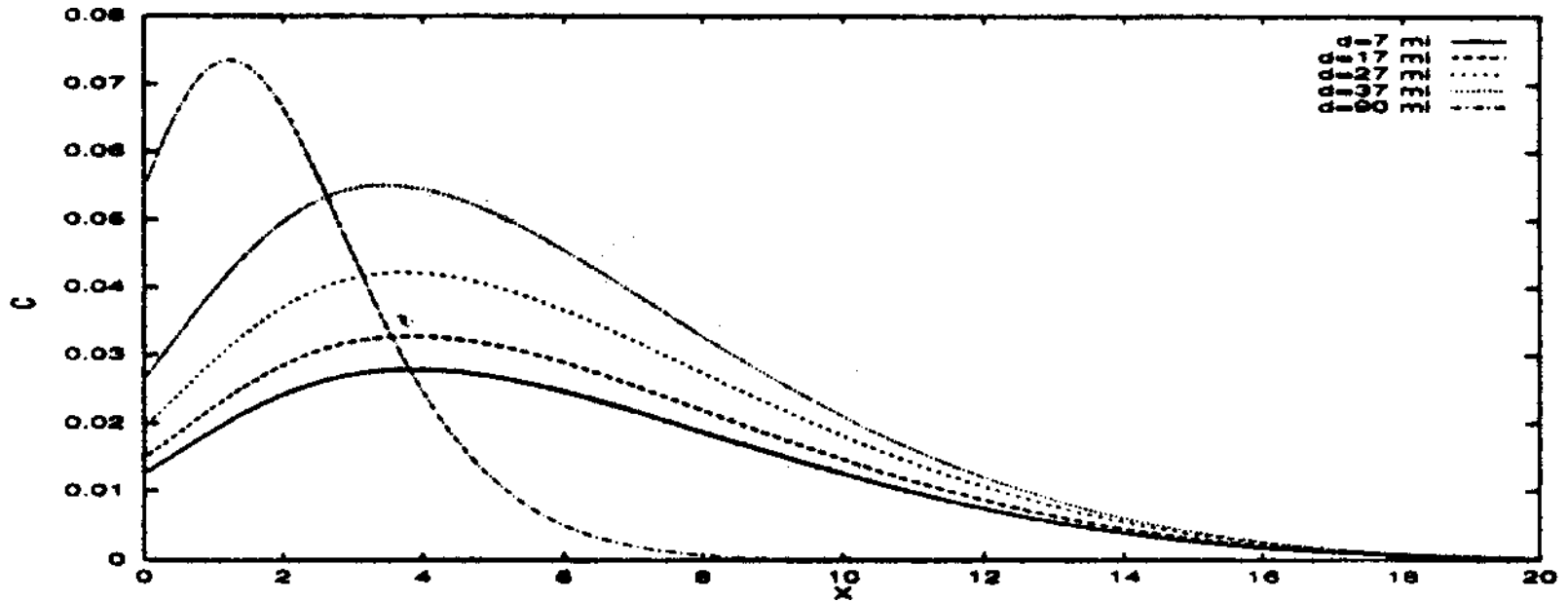
$$\phi = \theta(x^*, z^* - h^*, t^*) - \exp(v_f^* h^*) \theta(x^*, z^* + h^*, t^*),$$

Where

$$\theta(x^*, z^*, t^*) = (4\pi t^*)^{-1} \exp(-((x^* - t^*)^2 + (z^* + v_f^* t^*)^2) / (4t^*))$$

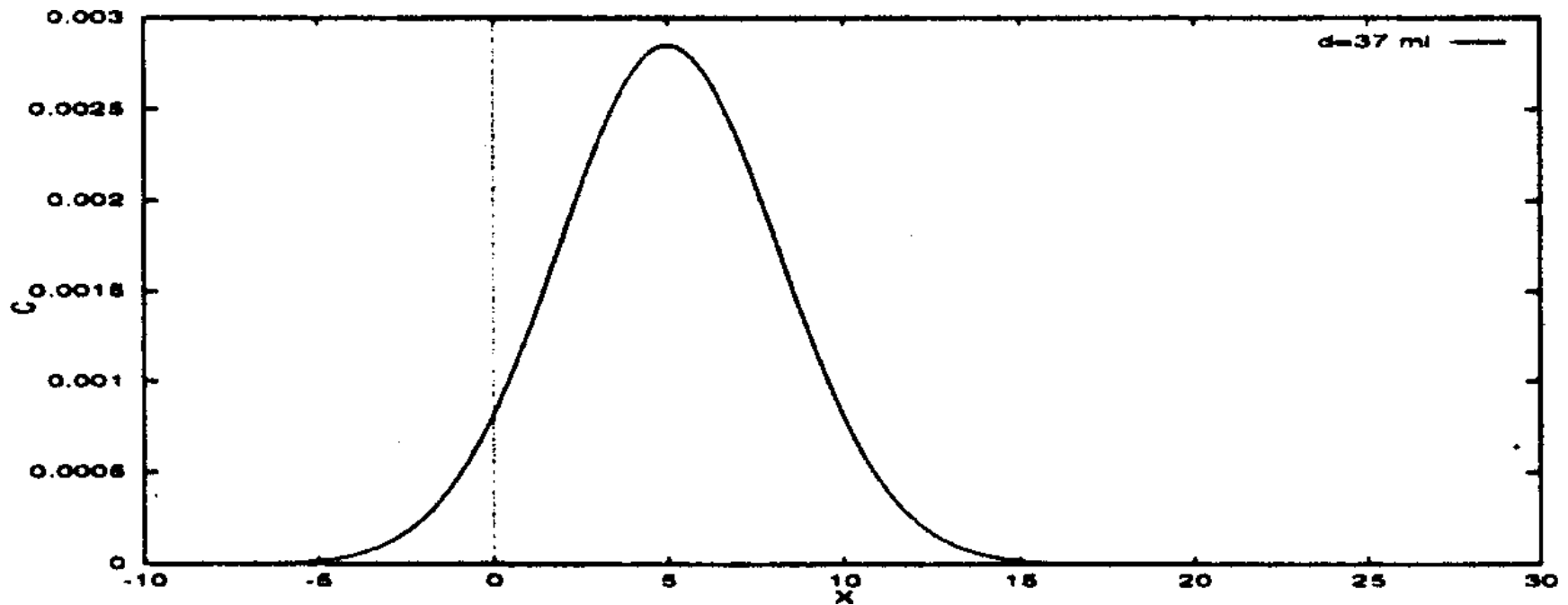
Numerical Experiments

Concentration of varying diameter particles 1m above ground. Wind constant at 2 m/sec and continuous source unit strength at 0.5m.



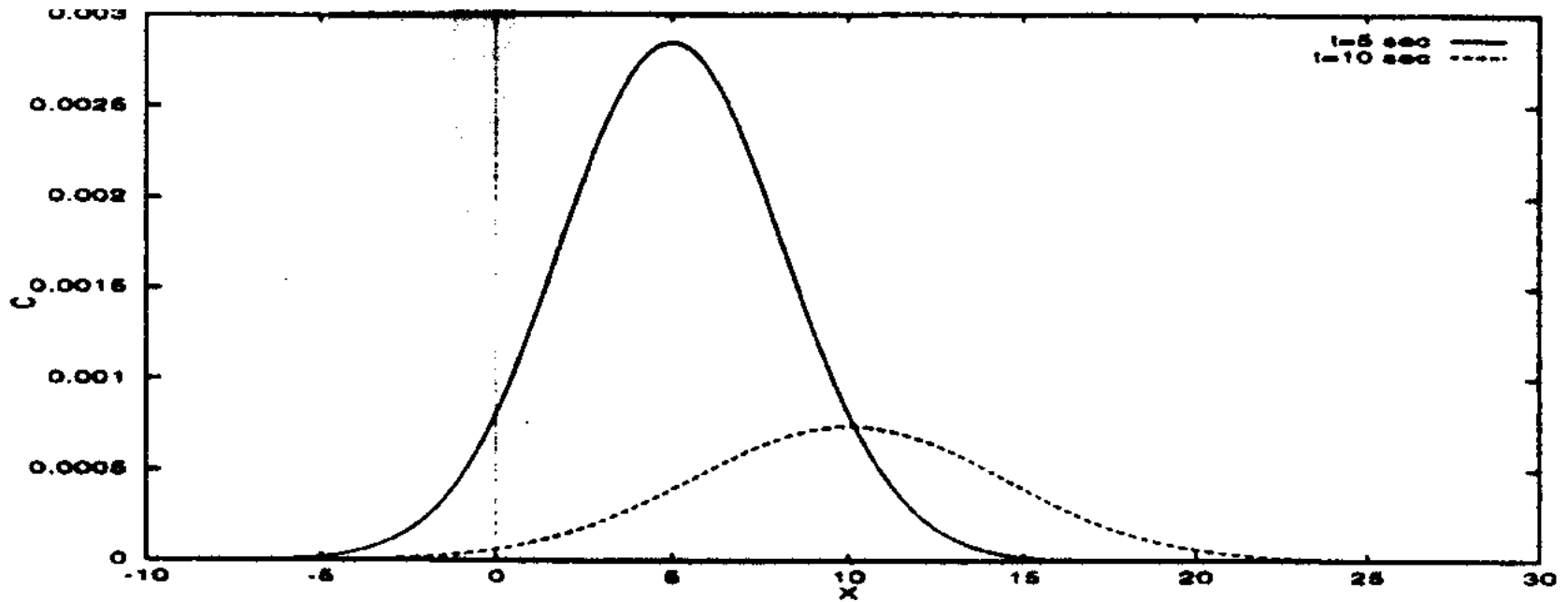
Particle Concentration for a Point Source

Concentration of 37 micron diameter particles 1m above ground after 5 seconds. Wind constant at 2 m/sec and continuous source unit strength at 0.5m.



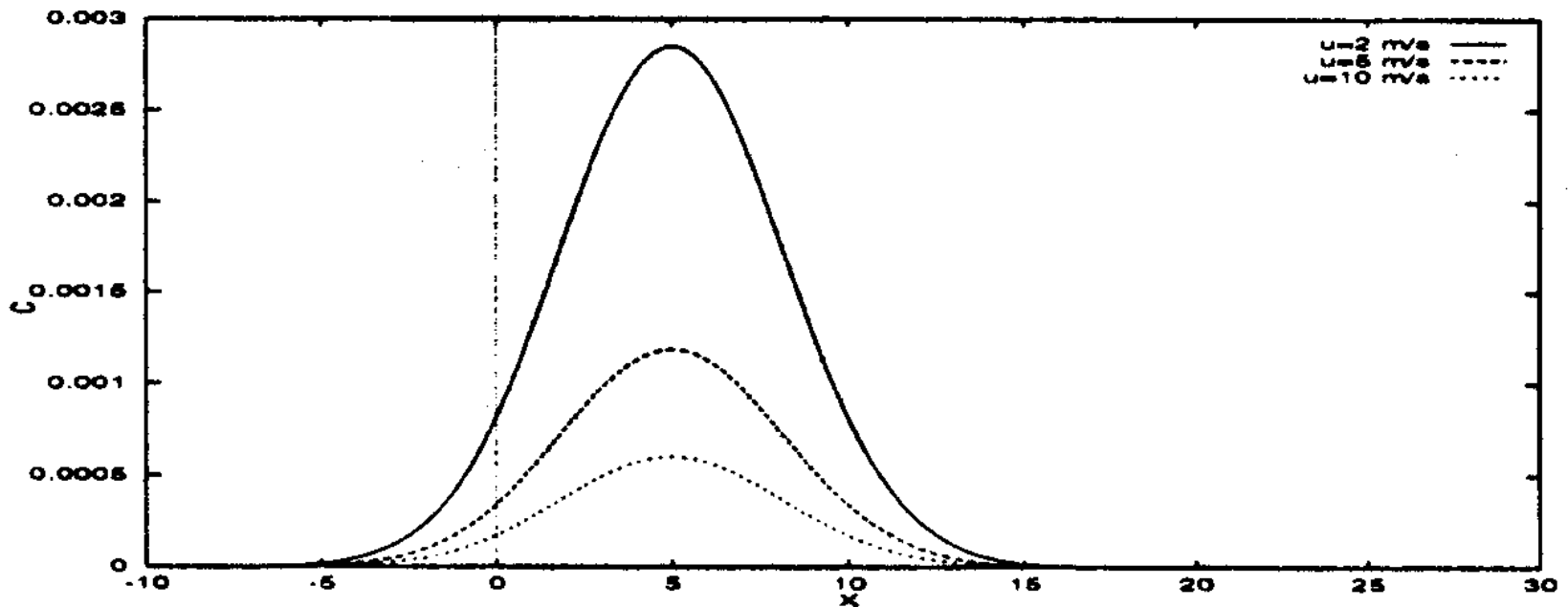
Data from Figure 3 after 5 and 10 Seconds

Particles travel at a constant speed, but the concentration drops.



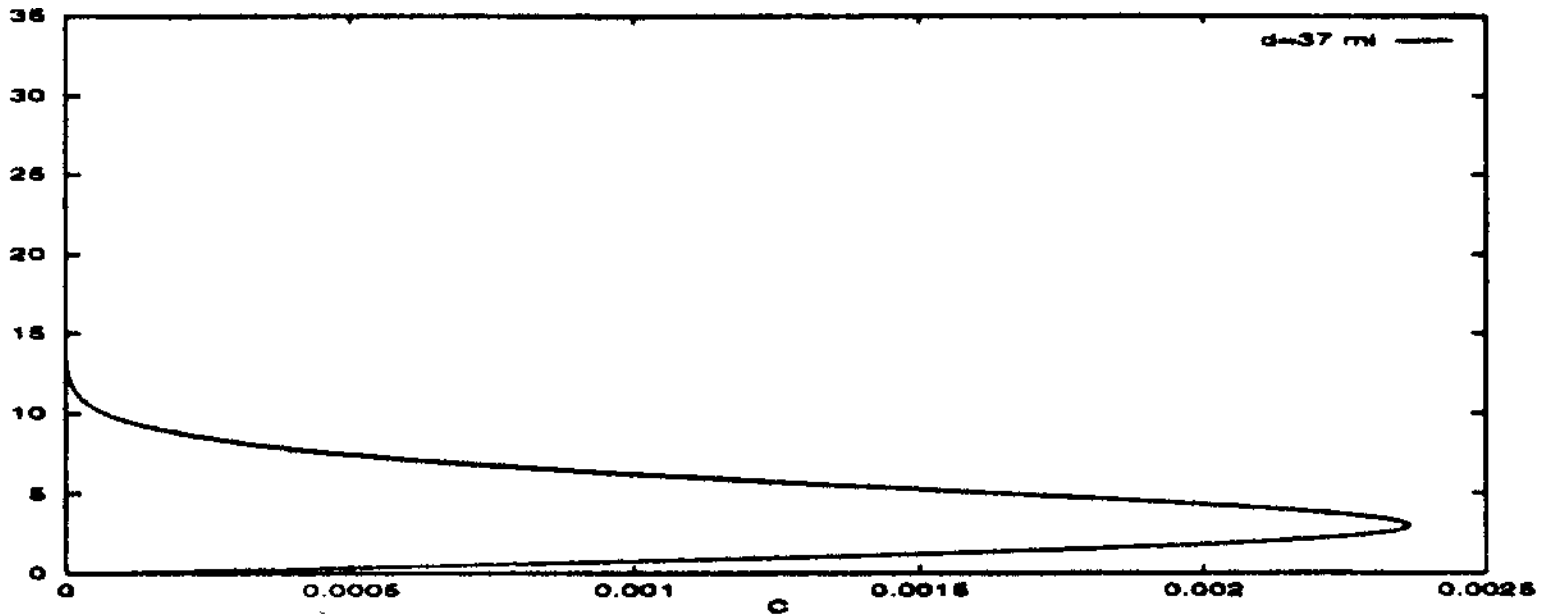
Different Wind Speeds

Concentration of 37 micron diameter particles 1m above ground after 5 seconds. Wind constant at 2, 5, and 10 m/sec and continuous source unit strength at 0.5m.



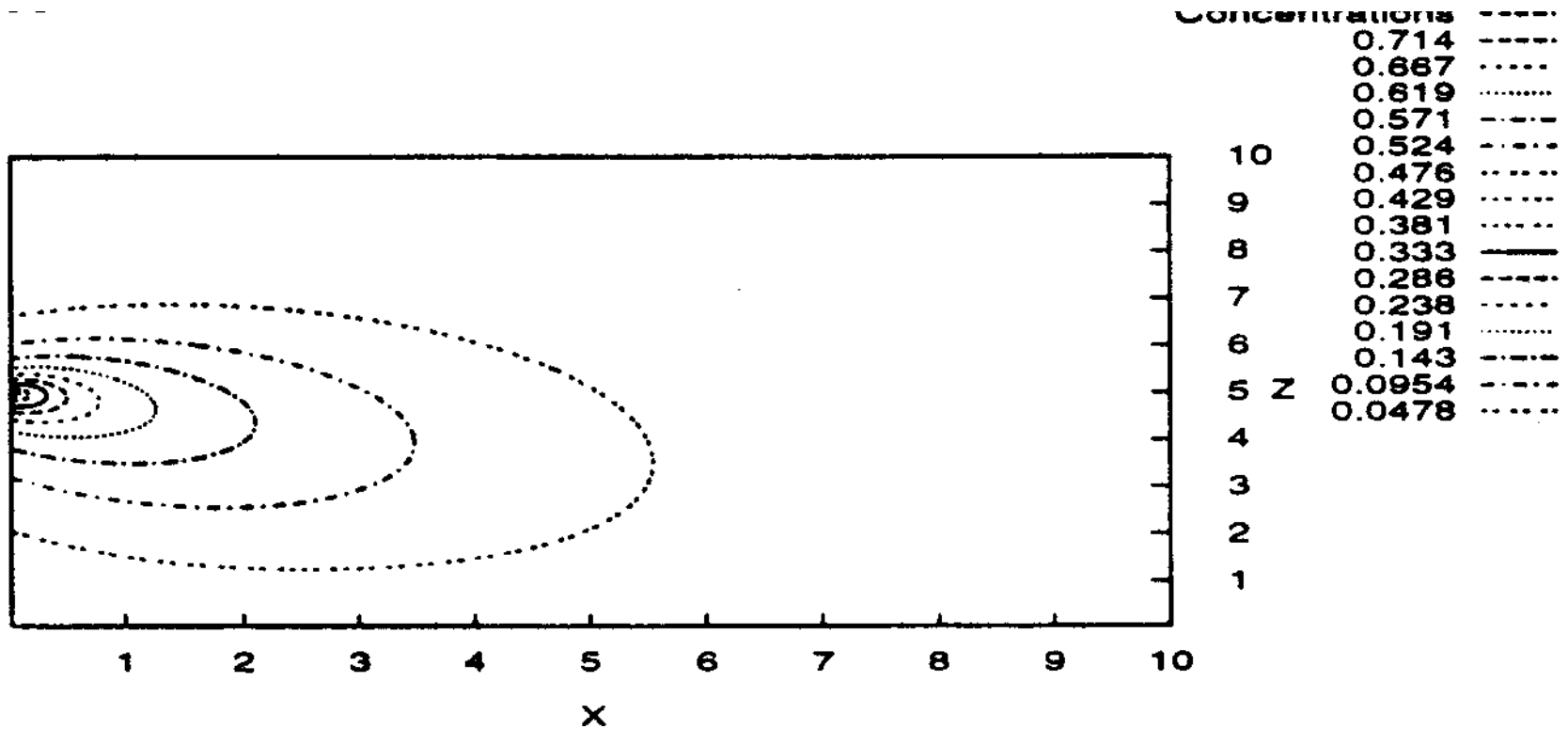
Particle Concentration at Different Heights

Concentration of 37 micron diameter particles above ground after 5 seconds. Wind constant at 2 m/sec and continuous source unit strength at 0.5m.



Particle Concentration Contours

Concentration of 37 micron diameter particles. Continuous source unit strength at 0.5m.



Possible Improvements

Nonconstants for things like D_x , D_z , ...

A Lagrangian approach instead of an Eulerian one

Individual particle motions are simulated by a Monte Carlo method

n bodies

Collisions of particles can be modeled

We can have more complicated initial distributions of particles (e.g., log, actual measurements, random, etc.)

Drawback: No analytic solution possible in general.

Different configurations

Faster velocities

Different shapes of particles

Multiple scales

Macro: turbulence

Micro: thermal agitation causing molecular movement

Deterministic and stochastic parts of the movement can be handled separately using affine spaces.