10th Anniversary International GAMM - Workshop on “Multigrid Methods”

Collection of Abstracts
version October 1, 1998

Abstracts

	Mon, Oct 5		Tue, Oct 6		Wed, Oct 7		Thu, Oct 8	
8:00 – 8:45	*Registration & Coffee*							
8:45 – 9:00	*Opening*							
	session chair: Langer		*session chair:* Braess		*session chair:* Hackbusch		*session chair:* Tobiska	
9:00 – 9:45	**Bank**		**T. Chan**		**Reusken**		**Korneev**	
9:45 – 10:15	**Brandt**		**J. Xu**		**Henson**		**Tyrtyshnikov**	
10:15 – 10:45	*Break*							
	CFD I *chair:* Schieweck	Mechanics I *chair:* Douglas	Physics *chair:* Denissenko	Mechanics II *chair:* Sauter	AMG *chair:* Kunoth	Convection I *chair:* Turek	Implementation *chair:* Witsch	CFD III *chair:* Vandewalle
10:45 – 11:15	**Dick**	**Gáspár**	**Ta'asan**	**Blaheta**	**Jung**	**Yeniceri**	**Douglas**	**F. Thiele**
11:15 – 11:45	**Fournier**	**Wieners**	**Livshits**	**Schöberl**	**Kickinger**	**Probst**	**Kowarschik**	**Hess**
11:45 – 12:15	**Hackenberg**	**Trofimov**	**Washio**	**Stiller**	**Fuhrmann**	**Le Borne**	**Zumbusch**	**Zhmakin**
12:15 – 12:45	*Lunch*		**Deuflhard**		*Lunch*			
12:45 – 14:00			*Lunch*					
14:00 – 14:30	**Yserentant**		*Excursion* boat trip to Linz Bonn–Linz 14:00–16:15 Linz–Bonn 17:20–18:45		**Stüben**		Nonlinear II *chair:* Manteuffel	Convection II *chair:* Kornhuber
	Homogenization *chair:* Fuhrmann	Wavelets I *chair:* Shaidurov			Helmholtz *chair:* Hiptmair	Nonlinear I *chair:* Tai	**Tai**	**Neuss**
14:30 – 15:00	**Sauter**	**Kunoth**			**Livshits**	**Kornhuber**	**Schulz**	**Stevenson**
15:00 – 15:30	**Bornemann**	**Harbrecht**			**M. Bader**	**Shaidurov**	**Starke**	**Pflaum**
15:30 – 16:00	*Break*				*Break*			
16:00 – 16:30	CFD II *chair:* Dick	Wavelets II *chair:* Stevenson			Maxwell *chair:* Livshits	Inverse *chair:* Starke	**Pasciak**	
	Reitzinger	**Quak**			**Hiptmair**	**Ressel**	Discretizations *chair:* Rüde	Convection III *chair:* Bornemann
16:30 – 17:00	**Schieweck**	**Urban**			**Kuhn**	**Henn**	**Lastdrager**	**Iliev**
17:00 – 17:30	**Turek**	**Hochmuth**			**Denissenko**	**Mohr**	**S. Schneider**	**John**
17:30 –							*Closing Remarks*	
	18:30 – *Reception*				19:00 – 20:00 *Visit the Arithmeum*			
					20:30 – *Conference Dinner*			

Participants

Program

Sunday, Oct 4.

Sun 19:00–21:00. *Registration* and come together at the restaurant ‘Bierhaus im Bären’, Acherstr. 1

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Monday, Oct 5.

Mon 8:00–8:45. *Registration & Coffee*.

Mon 8:45–9:00. *Opening* of the multigrid workshop.

Mon 9:00–10:15 , *session chair* U. Langer.

Mon 9:00–9:45 , Randolph E. Bank. *Multigraph Iterative Methods*.

Mon 9:45–10:15 , Achi Brandt. *Achieving Textbook Multigrid Efficiency (TME) in CFD*.

Mon 10:15–10:45. *Coffee Break*.

Mon 10:45–12:15 , CFD I *session chair* F. Schiewck.

Mon 10:45–12:15 b, Mechanics I *session chair* C. Douglas.

Mon 10:45–11:15 , J. Vierendeels, K. Riemslagh, E. Dick. *A multigrid semi-implicit line method for Navier-Stokes equations*.

Mon 10:45–11:15 b, Csaba Gáspár. *Quadtree grids and some applications in flow problems*.

Mon 11:15–11:45 , Luc Fournier, Stéphane Lanteri. *An additive multigrid method using a filtering concept to solve flow simulations on parallel computers*.

Mon 11:15–11:45 b, Christian Wieners. *Multigrid methods for stabilized finite elements for elasticity and the application to Prandtl-Reuß-plasticity*.

Mon 11:45–12:15 , U. Becker-Lemgau, M. G. Hackenberg, B. Steckel, R. Tilch. *Multigrid for Multidisciplinary Applications: Strip Steel Production*.

Mon 11:45–12:15 b, Alexander V. Trofimov. *Multigrid Methods for Elasto-Plastic Problems with the Stress Concentration*.

Mon 12:15–14:00. *Lunch* at the university club (free)

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Mon 14:00–15:30 , Homogenization, *session chair* J. Fuhrmann.

Mon 14:00–14:30 , Harry Yserentant. *Coarse grid spaces for domains with a complicated boundary*.

Mon 14:30–15:30 b, Wavelets I, *session chair* V. Shaidurov.

Mon 14:30–15:00 , Stefan Sauter. *Multi-grid methods for PDEs on complicated domains*.

Mon 14:30–15:00 b, Angela Kunoth. *On the Treatment of Bounded Domains and Boundary Conditions in Adaptive Wavelet Methods*.

Mon 15:00–15:30 , Folkmar Bornemann. *A New Algorithmic Approach to Multigrid-Homogenization*.

Mon 15:00–15:30 b, Helmut Harbrecht. *Construction of Globally Continuous Biorthogonal Wavelet Bases on Domains in* $\mathbb{R}^2$.

Mon 15:30–16:00. *Coffee Break*.

Mon 16:00–17:30 , CFD II, *session chair* E. Dick.

Mon 16:00–17:30 b, Wavelets II, *session chair* R. Stevenson.

Mon 16:00–16:30 , Gundolf Haase, Stefan Reitzinger. *Robust Algebraic Multigrid Methods in Magnetic Shield Problems*.

Mon 16:00–16:30 b, Michael S. Floater, Ewald G. Quak. *Piecewise Linear Prewavelets on Arbitrary Triangulations*.

Mon 16:30–17:00 , Friedhelm Schieweck. *Multigrid Methods for Higher Order Discretizations of the Navier–Stokes Equations*.

Mon 16:30–17:00 b, Stephan Dahlke, Reinhard Hochmuth, Karsten Urban. *Adaptive Multiscale Methods for Saddlepoint Problems*.

Mon 17:00–17:30 , Stefan Turek. *Robust multigrid for edge-oriented discretizations*.

Mon 17:00–17:30 b, Reinhard Hochmuth. *Restricted Nonlinear Approximation and Applications*.

Mon 18:30–. *Reception* at the old town hall (free)

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Tuesday, Oct 6.

Tue 9:00–10:15 , *session chair* D. Braess.

Tue 9:00–9:45 , Tony Chan. *An Energy-Minimizing Approach to Robust Multigrid Methods*.

Tue 9:45–10:15 , Jinchao Xu, Aihui Zhou. *Some Local/Parallel Algorithms for Nonlinear Elliptic Equations*.

Tue 10:15–10:45. *Coffee Break*.

Tue 10:45–11:15 , Byungduck Chough, Shlomo Ta’asan. *From Molecular Dynamics to Continuum Models: A Numerical Approach*.

Tue 10:45–11:15 b, Radim Blaheta. *Composite grid solvers for elasticity and plasticity problems*.

Tue 11:15–11:45 , Ira Livshits, Shlomo Ta’asan. *From Stochastic Lattice Dynamics Models to Partial Differential Equations*.

Tue 11:15–11:45 b, Joachim Schöberl. *Robust Multigrid Methods for Parameter Dependent Problems*.

Tue 11:45–12:15 , Takumi Washio, C. W. Oosterlee. *Error analysis for a potential problem on locally refined grids*.

Tue 11:45–12:15 b, Jörg Stiller, Krzysztof Boryczko, Wolfgang E. Nagel. *PML - a parallel multilevel system for unstructured grids*.

Tue 12:15–12:45 , Peter Deuflhard. *Multigrid FEMs in Clinical Cancer Therapy Planning*.

Tue 12:45–14:00. *Lunch* at the university club (free).

Tue 14:00–. *Excursion:* boat trip on the river Rhine to the pitoresk city Linz (fee included). the boat departs from the ‘Alter Zoll’ at 14:00. stay in Linz 16:15–17:20. arrival in Bonn at 18:45.

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Wednesday, Oct 7.

Wed 9:00–10:15 , *session chair* W. Hackbusch.

Wed 9:00–9:45 , J. Bey, A. Reusken. *On the convergence of basic iterative methods for convection-diffusion problems*.

Wed 9:45–10:15 , Andrew J. Cleary, Robert D. Falgout, Van Emden Henson, Jim E. Jones. *Parallel Coarse-Grid Selection and Element Interpolation for Algebraic Multigrid*.

Wed 10:15–10:45. *Coffee Break*.

Wed 10:45–11:15 , Michael Jung. *Parallel Algebraic Multilevel Iteration Methods*.

Wed 10:45–11:15 b, Erkan Yeniceri, Mine Kacan, Ali Ecder, Levent Ertoz, Bahadir Mertan. *Application of multigrid methods to 2-D MHD Hartmann flow*.

Wed 11:15–11:45 , Ulrich Langer, Ferdinand Kickinger. *Algebraic Multigrid based on Graph Coarsening*.

Wed 11:15–11:45 b, Thomas Probst. *On ordering strategies for multigrid methods*.

Wed 11:45–12:15 , Jürgen Fuhrmann. *Variants of modular algebraic multigrid methods*.

Wed 11:45–12:15 b, Sabine Le Borne. *Ordering techniques for convection dominated problems on unstructured three-dimensional grids*.

Wed 12:15–14:00. *Lunch* at the university club (free)

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Wed 14:00–14:30 , Klaus Stüben, Arnold Krechel. *Parallel Algebraic Multigrid*.

Wed 14:30–15:00 , Achi Brandt, Ira Livshits. *Multigrid Wave-Ray Algorithms for Helmholtz Equations*.

Wed 14:30–15:00 b, Ralf Kornhuber. *On monotone iterations for variational inequalities*.

Wed 15:00–15:30 , Christoph Zenger, Michael Bader. *Hierarchical Bases for the Indefinite Helmholtz Equation*.

Wed 15:00–15:30 b, V. V. Shaidurov. *Cascade iterative algorithms in finite element method for non-linear elliptic equations*.

Wed 15:30–16:00. *Coffee Break*.

Wed 16:00–16:30 , Ralf Hiptmair. *Multigrid in* $H(\mathrm{div})$ *and* $H(\mathbf{curl})$.

Wed 16:00–16:30 b, Klaus J. Ressel. *A Multilevel approach for the retrieval of atmospheric trace gases*.

Wed 16:30–17:00 , M. Kuhn. *Multigrid Methods for Magnetic Field Problems*.

Wed 16:30–17:00 b, Kristian Witsch, Stefan Henn. *A Multigrid Approach For Minimizing A Nonlinear Functional For Digital Image Matching*.

Wed 17:00–17:30 , Valeri V. Denissenko. *Multigrid method for a global conductor in the Earth’s ionosphere*.

Wed 17:00–17:30 b, Marcus Mohr, Constantin Popa, Ulrich Rüde. *Multigrid Methods for an Inverse Potential Problem*.

Wed 19:00–20:00. *Guided tour through the Arithmeum* of the Institute for Discrete Mathematics (Prof. B. Korte) (free). Lennéstr. 2, A museum of historic, mechanic computing machinery along with modern electronic computers.

Wed 20:30–. *Conference Dinner* at the restaurant ‘Em Höttche’, Am Markt 4 (meal is free)

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Thursday, Oct 8.

Thu 9:00–10:15 , *session chair* L. Tobiska.

Thu 9:00–9:45 , Vadim Korneev. *Recent developments in Schwarz algorithms for the h-p-version of the finite element method*.

Thu 9:45–10:15 , Eugene Tyrtyshnikov. *Theory and Applications of Mosaic-Skeleton Method*.

Thu 10:15–10:45. *Coffee Break*.

Thu 10:45–11:15 , Craig C. Douglas, Jonathan Hu, Marco Bittencourt. *Cache Based Multigrid on Quasi-Structured and Unstructured Grids*.

Thu 10:45–11:15 b, J. Yan, L. Xue, F. Thiele. *A Modified Full Multigrid Algorithm For Navier-Stokes Equations*.

Thu 11:15–11:45 , Ulrich Rüde, Markus Kowarschik. *Cache-aware Multigrid for 3D Elliptic Equations*.

Thu 11:15–11:45 b, Reinhold Hess. *Dynamically Adaptive Multigrid on Parallel Computers for a Semi–Implicit Discretization of the Shallow Water Equations*.

Thu 11:45–12:15 , Michael Griebel, Gerhard Zumbusch. *Key Based Multigrid on Adaptive Grids*.

Thu 11:45–12:15 b, D. O. Ofengeim, S. K. Kochuguev, A. I. Zhmakin. *Multigrid Methods for Low Mach Number Viscous Flows on Adaptive Unstructured Grids*.

Thu 12:15–14:00. *Lunch* at the university club (free)

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Thu 14:00–14:30 , Xue-Cheng Tai, Jinchao Xu. *Nonlinear space decomposition for degenerated and singular nonlinear equations and some asyncronous versions*.

Thu 14:00–14:30 b, Nicolas Neuss. *Adaptive Multigrid Solving of Large Systems of Chemical Reactions with Diffusion and Transport*.

Thu 14:30–15:00 , Volker Schulz. *Solving Optimal Control Problems by Multigrid Methods with Transforming Smoothers*.

Thu 14:30–15:00 b, Rob Stevenson. *An efficient multigrid method for the Morley discretization of the biharmonic equation*.

Thu 15:00–15:30 , Gerhard Starke. *Least-Squares Mixed Finite Elements for a Nonlinear Elliptic Problem: A Gauss-Newton Multilevel Method*.

Thu 15:00–15:30 b, Christoph Pflaum. *Construction of Robust Multilevel Splittings*.

Thu 15:30–16:00. *Coffee Break*.

Thu 16:00–16:30 , Joseph E. Pasciak. *Multigrid for the mortar finite element method*.

Thu 16:30–17:00 , Boris Lastdrager, Barry Koren. *Error Analysis for Sparse-Grid Recombination*.

Thu 16:30–17:00 b, D.Drikakis, O.Iliev, D.Vassileva. *An adaptive smoothing-based multigrid algorithm for flow computations*.

Thu 17:00–17:30 , Stefan Schneider, Christoph Zenger. *Multigrid Methods for Hierarchical Adaptive FE*.

Thu 17:00–17:30 b, Volker John, Lutz Tobiska. *Smoothers in Coupled Multigrid Methods for the Stokes and Navier–Stokes Equations*.

Thu 17:30–. *Closing remarks*.

In Cooperation with the

- GAMM-Committee “Discretization Methods in Solid Mechanics”
- GAMM-Committee “Efficient Numerical Methods for PDEs”
- SFB 256 ,,Nichtlineare Partielle Differentialgleichungen“

Programme Committee

- Dietrich Braess (Bochum, Germany)
- Michael Griebel (Bonn, Germany)
- Wolfgang Hackbusch (Kiel, Germany)
- Ulrich Langer (Linz, Austria)

Local Organizing Committee

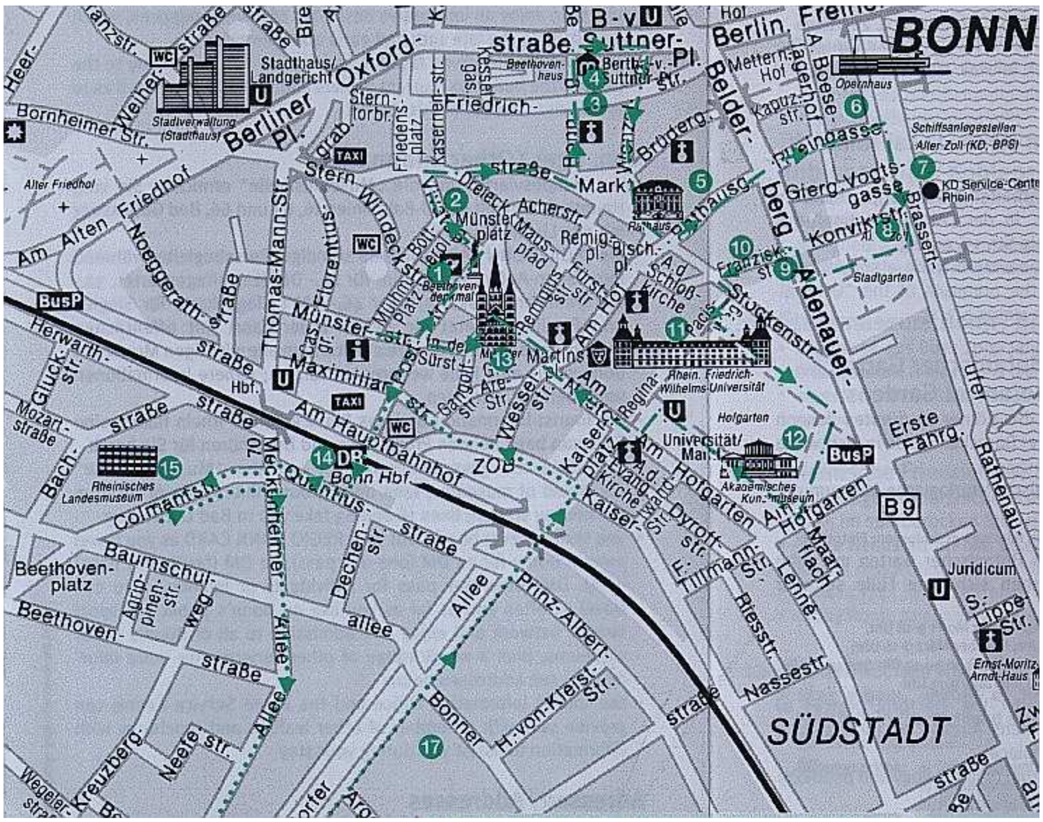
- Michael Griebel (griebel@iam.uni-bonn.de)
- Frank Kiefer (kiefer@iam.uni-bonn.de)
- Gerhard Zumbusch (zumbusch@iam.uni-bonn.de)

Special thanks for the guided tour through the Arithmeum of the Institute for Discrete Mathematics (Prof. B. Korte).

From Monday to Thursday during the workshop, the conference desk can be reached via phone and via Fax at ++49-228-7296116. Before and after the workshop, please use email `mg10@iam.uni-bonn.de`, phone ++49-228-733427 and Fax ++49-228-737527.

Computing facilities for reading email etc. are located at the Institute, Wegelerstr. 6, 1st floor, rooms 113/114, IAM–CIP. During the workshop there are guest accounts available.

The workshop is hold at the University Club



Universitätsclub
Konviktstraße 9
D-53113 Bonn
Germany

Train: Getting off the train at the Bonn Central Railway station you take the main exit in direction downtown and walk via ’Kaiserplatz’ to the ’Hofgarten’ which is the park behind the baroque (yellow) University Main Building. From there you walk along the University Main Buliding to the ’Koblenzer Tor’ cross the ’Konrad Adenauer Allee’ and turn directly right which is already the ’Konviktstrasse’ where the University Club is located.

Underground: The nearest underground station is the Bonn Universität/Markt station. Getting off the underground there you take the exit ’Markt’ and cross the ’Konrad Adenauer Allee’. From there you walk through the ’Koblenzer Tor’ and turn directly right which is already the ’Konviktstrasse’ where the University Club is located.

An alternative is getting off the underground at the Bonn Central Railway station and following the description to the University Club from there.

Bus: All buses will finaly stop at the Bonn Central Bus station which is located directly in front of the Bonn Central Railway station. So take simply the description for getting to the University Club arriving by train.

By car: Parking in and around the conference centre is limited and expensive. The best thing is to park your car somewhat outside or next to your hotel and use public transport.

Travel by air: The closest airport is Cologne/Bonn with easy connections to Bonn. To get to Bonn from the Cologne/Bonn Airport you can:

take a taxi: ca. DM 55,– , 20-30 min.

take the bus: Bus 670 (time table) from the airport to the central railway station in Bonn, (every 20 minutes, DM 8.30, 35-45 min). Follow the description to the University Club from there.

1 Algebraic Methods

1.1 Multigraph Iterative Methods

Mon 9:00–9:45

Randolph E. Bank
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The class of Multigraph Iterative Methods is obtained by examining the close connection between classical multigrid and hierarchical basis methods and incomplete LU decomposition. Using this connection, one is able to formally generalize both hierarchical basis and multigrid methods to the case of general sparse matrix graphs. These correspond to ILU methods in which certain fill-in edges are allowed. In this talk, I will discuss the derivation of the methods, and present some recent numerical results.

1.2 An Energy-Minimizing Approach to Robust Multigrid Methods

Tue 9:00–9:45

Tony F. Chan
UCLA, USA
e-mail chan@math.ucla.edu

Joint work with Barry Smith and W. L. Wan

We describe a new approach to construct robust interpolation operators for use within multigrid methods, which can handle in a unified fashion problems with problematic (e.g. anisotropic , discontinuous or oscillatory) coefficients, as well as for non-nested unstructured grids. The basic idea derives from recent domain decomposition theory and is based on defining coarse basis functions (from which the interpolation operators can be easily derived) which are *stable* (minimize the total energy in a global sense) and have good approximation properties (preserving constants). Numerical results will be presented which show that the resulting multigrid method are very effective.

1 Presented by Tony F. Chan

1.3 Parallel Coarse-Grid Selection and Element Interpolation for Algebraic Multigrid

Wed 9:45–10:15

Andrew J. Cleary
Robert D. Falgout
Van Emden Henson
Jim E. Jones
Lawrence Livermore National Laboratory,
Center for Applied Scientific Computing
email: vhenson@llnl.gov

The need to solve linear systems arising from problems posed on extremely large, unstructured grids has sparked great interest in parallelizing algebraic multigrid (AMG). The ”classical” AMG algorithm, however, selects the coarse-grid points in a manner that is inherently sequential, and no scalable way of parallelizing it is known. Unless scalable parallel coarse-grid selection algorithms are developed, coarse-grid selection could become a significant fraction of the total effort, even to the point of dominating the computation time.

In this talk we examine two important concepts. The first is the formulation of a parallel coarsening algorithm based on heuristics designed to insure coarse-grid quality by honoring the relationships of the independent variables to each other. This new technique is similar to the standard approach, but the heuristics used are modified to permit parallel processing. Experimental results are described.

The second concept is a careful examination of the fact that the underlying requirement for the coarse grid is to accurately represent ”smooth” errors that are not reduced by relaxation. Further, the prolongation operator must be designed to transfer these components accurately to the fine grid. By using information from the finite element stiffness matrices, we develop a theory that produces the ”optimal” prolongation for a given coarse grid, and we explore methods of using this information to select high-quality coarse grids.

1 Presented by Van Emden Henson

1.4 Parallel Algebraic Multilevel Iteration Methods

Wed 10:45–11:15

Michael Jung
Fakultät für Mathematik, Technische Universität Chemnitz, Germany
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In recent years, highly efficient iterative solvers have been developed for large scale systems of linear equations resulting from finite element discretizations of elliptic boundary value problems. Examples for such solvers are the multigrid methods and the preconditioned conjugate gradient (PCG) method with additive multilevel preconditioners, with algebraic multilevel iteration preconditioners, or with preconditioners based on multigrid methods.

In the talk we will discuss some variants of algebraic multilevel iteration (AMLI) methods for elliptic boundary value problems in two- and three-dimensional domains. Numbering first the nodes of the mesh $\mathcal{T}_{l-1}$ and then the nodes of the finer mesh $\mathcal{T}_l$, the finite element stiffness matrix A_l has the block structure

$$\begin{pmatrix} A_{l,vv} & A_{l,vm} \\ A_{l,vm}^T & A_{l,mm} \end{pmatrix} = \begin{pmatrix} A_{l,vv} - A_{l,vm} A_{l,mm}^{-1} A_{l,vm}^T & A_{l,vm} \\ 0 & A_{l,mm} \end{pmatrix} \begin{pmatrix} I_{l,v} & 0 \\ A_{l,mm}^{-1} A_{l,vm}^T & I_{l,m} \end{pmatrix} .$$

An AMLI preconditioner derived from this factorization of the stiffness matrix has the block structure

$$C_l = \begin{pmatrix} \tilde{C}_{l-1}^{\mathrm{C}} & \tilde{C}_{l,vm} \\ 0 & C_{l,mm} \end{pmatrix} \begin{pmatrix} I_{l,v} & 0 \\ C_{l,mm}^{-1} \tilde{C}_{l,vm}^T & I_{l,m} \end{pmatrix} ,$$

where $\tilde{C}_{l,vm} = A_{l,vm} + J_{l,mv}^T (A_{l,mm} - C_{l,mm})$, $(\tilde{C}_{l-1}^{\mathrm{C}})^{-1}$ is defined by $(\tilde{C}_{l-1}^{\mathrm{C}})^{-1} = (I_{l-1} - P_\mu(C_{l-1}^{-1} A_{l-1})) A_{l-1}^{-1}$ (P_μ being a polynomial of degree μ), and $C_{l,mm}$ approximates the block $A_{l,mm}$ of the stiffness matrix A_l. C_{l-1} is defined analogously to C_l, and $C_1 = A_1$ (see, e.g., AXELSSON and VASSILEVSKI 1989, 1990, 1992).

The main ingredients of the AMLI preconditioner C_l are the matrix $C_{l,mm}$ and the polynomial $P_\mu(t)$. We define the matrix $C_{l,mm}$ implicitly by the application of iterative solvers, e.g. the Jacobi method or the symmetric Gauss-Seidel method. Polynomials of the type $P_\mu(t) = (1 - t)^\mu$ or polynomials derived from shifted Chebyshev polynomials will be used for the definition of $(\tilde{C}_{l-1}^{\mathrm{C}})^{-1}$.

We discuss convergence properties of the AMLI-PCG methods and the implementation of these methods on parallel computers with MIMD architecture. Finally, we compare multigrid preconditioners, additive multilevel preconditioners and AMLI preconditioners on numerical examples.

1.5 Algebraic Multigrid based on Graph Coarsening

Wed 11:15–11:45

Ulrich Langer
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Ferdinand Kickinger
University Linz Austria
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In the numerical simulation of industrial problems fast solvers are needed, especially when dealing with complicated geometries in 3D arising from real-life problems. Multigrid (MG) Methods are quite useful in this context, because of their optimal time complexity. One of the major problems caused by the use of MG, is the sequence of nested meshes needed. This sequence is often not available or reconstructible in a natural manner. To overcome this crucial point, we generalize some Algebraic Multigrid Method (AMG) for scalar elliptic equations to systems of equations. This method is based on a graph coarsening algorithm acting on the stiffness pattern, and therefore, it is cheap. From this point of view, the standard MG with Galerkin projection turns out to be some special case of our algebraic algorithm.

Industrial problems often cause further trouble based on some bad scaled property. Therefor, engineers use the so-called Global Extraction Element-by-Element (GE-EBE) Methods, The authors propose to use the GE-EBE methods and their patch counterpart (GE-PBP) either as smoothers, especially, in their multiplicative version, or as preconditioners in their multilevel additive version. The GE-PBP smoother is especially suited for the use in Algebraic Multigrid Methods based on graph coarsening strategies. The numerical results presented confirm the good smoothing properties of GE-PBP smoothers as well as the preconditioning properties of multilevel additive GE-PBP preconditioners. Finally, the authors discuss Algebraic Multigrid Methods for the Stokes Problem using Uzawa typed methods.

1 Presented by Ferdinand Kickinger

1.6 Variants of modular algebraic multigrid methods

Wed 11:45–12:15

Jürgen Fuhrmann
WIAS Berlin, Germany
e-mail fuhrmann@wias-berlin.de

Flexibility of use is essential for the usefulness of a numerical method for applied projects. Thus we favorize a modularizable algorithmic approach to algebraic multigrid methods. It is based on the matrix graph and at various stages of the solution process different parts of the multigrid preconditioner are established.

Based on numerical experiments, we compare the two main approaches to this aim, namely the vertex centered approach with matrix dependent transfer operators and the cell centered approach with piecewise constant interpolations. We discuss issues like numerical stability and efficiency of the implementation.

1.7 Parallel Algebraic Multigrid

Wed 14:00–14:30

Klaus Stüben , Arnold Krechel
GMD/SCAI, Germany
email: klaus.stueben@gmd.de

In contrast to geometric multigrid methods, in algebraic multigrid (AMG) methods the construction of a hierarchy of coarser levels, including the corresponding transfer operators, is part of the algorithm. The information required for an automatic coarsening is taken from the given finest-level matrix. For various types of matrices, this approach has proven to be robust, efficient and very flexible. In particular, AMG can directly be applied to a wide range of discretized elliptic PDEs on unstructured grids, both in 2D and 3D.

Since the hierarchy of coarser levels and the related operators develop dynamically during the setup phase of an AMG algorithm, it is quite difficult to parallelise AMG efficiently. A ”native” parallelisation would, in general, require unpredictable and highly complex communication patterns which seriously limit the achievable efficiency, in particular of the setup phase. In addition, the original AMG approach contains inherently sequential algorithmic components to construct the coarser levels.

A parallelisation approach is presented which limits the communication without sacrificing convergence in complex situations. Results will be presented for industrial CFD applications on various parallel machines.

1 Presented by Klaus Stüben

1.8 Theory and Applications of Mosaic-Skeleton Method

Thu 9:45–10:15

Eugene Tyrtyshnikov
Institute of Numerical Mathematics, Russian Academy of Sciences,
Gubkina 8, Moscow 117333, Russia
e-mail: tee@bach.inm.ras.ru)

The mosaic-skeleton method was bred in a simple observation that rather large blocks in very large matrices coming from integral formulations can be approximated accurately by a sum of just few skeletons (some say dyads or rank-one matrices). These blocks might correspond to a region where the kernel is smooth enough, and anyway it can be a region where the kernel is approximated by a short sum of separable functions (in other words, functional skeletons). The mosaic-skeleton approximations are easy to result in fast approximate matrix-vector multiplication algorithms close by nature to those of multipole, interpolation, and wavelet-based approaches. All the said techniques involve some hierarcy of interface regions and function approximants. What the mosaic-skeleton method differs in from others is a matrix analysis view on largely the same problem. Such a view can be very useful due to the generality of matrix theory approaches. Since the effect of approximations is like that of having small-rank matrices, we find it pertinent to say about *mosaic ranks* of a matrix which turn to be pretty small for many nonsingular matrices.

We cover a wide class of applications. In particular, following Brandt, we propose to call $f(x, y)$ an *asymptotically smooth* function if there exist $c, d > 0$ and a real number g such that

$$|\partial^p f(x, y)| \ \leq \ c\, d^p\, p!\, |x - y|^{g-p},$$

where ∂^p is any p-order derivative in y. Then, for $n\times n$ matrices $A_n = [f(x_i^{(n)}, y_j^{(n)}]$ with quasiuniform meshes $\{x_i^n\}$ and $\{y_j^{(n)}\}$ in a bounded m-dimensional domain, we prove that the approximate mosaic ranks might grow only logarithmically in n. The results obtained lead immediately to $O\left(n \log n\right)$ matrix-vector multiplication algorithms. Using a special technique for harmonic kernels, we can sharpen the estimates. A likely technique can be used for oscillatory (and hence not asymptotically smooth) kernels.

General matrix approximation algorithms were applied to integral equations of lots of applications from the classical potential flow or electrostatic problems to thermal analysis for stratified media and electromagnetic scattering problems. Selected numerical results will be presented.

2 CFD

2.1 Achieving Textbook Multigrid Efficiency (TME) in CFD

Mon 9:45–10:15

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”Textbook multigrid efficiency” means solving a discrete PDE problem in a computational work which is only a small (less than 10) multiple of the operation count in the discretized system of equations itself. As a road map for attaining this optimal performance for general CFD problems, we list in a table every foreseen kind of computational difficulty for achieving that goal, together with the possible ways for resolving that difficulty, their current state of development, and references.

Included in the table are staggered and nonstaggered, conservative and non-conservative discretizations of viscous and inviscid, incompressible and compressible flows at various Mach numbers, as well as a simple (algebraic) turbulence model and comments on chemically reacting flows. The listing of associated computational barriers involves: non-alignment of streamlines or sonic characteristics with the grids; recirculating flows; stagnation points; discretization and relaxation on and near shocks and boundaries; far-field artificial boundary conditions; small-scale singularities; large grid aspect ratios; boundary layer resolution; and grid adaption.

2.2 A multigrid semi-implicit line method for Navier-Stokes equations

Mon 10:45–11:15

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Preconditioning of the incompressible and compressible Navier-Stokes equations is used by many authors in order to accelerate convergence, especially for low Mach number flow. However this technique does not always provide good results on high aspect ratio grids, because of the stiffness due to the numerically-anisotropic behaviour of the diffusive and acoustic terms.

The stiffness on high aspect ratio grids is due to two reasons : first, the discretized diffusion terms are numerically very anisotropic on such grids and second, the ratio of the cfl numbers in both directions is proportional to the aspect ratio. Assume a control volume with Δy much smaller than Δx. For incompressible flow, using an artificial compressibility approach, the timestep $\Delta\tau$ can be computed as

$$\Delta\tau_e = \left(\frac{u+c_x}{\Delta x}+\frac{v+c_y}{\Delta y}\right)^{-1} \quad \text{or} \quad \Delta\tau_i = \left(\frac{u+c_x}{\Delta x}+\frac{v}{\Delta y}\right)^{-1}$$

for explicit and line-implicit methods respectively, with $c_x = \sqrt{(u^2+\beta^2)}$ and $c_y = \sqrt{(v^2+\beta^2)}$ and β the pseudo-acoustic speed. Let’s define the convective and acoustic cfl numbers in x- and y-direction as $cfl_{xc} = (u\Delta\tau)/\Delta x$, $cfl_{yc} = (v\Delta\tau)/\Delta y$, $cfl_{xa} = (u + c_x)\Delta\tau/\Delta x$, $cfl_{ya} = (v + c_y)\Delta\tau/\Delta y$. If the flow is aligned to the x-direction, and high aspect ratios are used, and with $\Delta\tau = \Delta\tau_e$ and $cfl_{ya} = 1$, cfl_{xc} becomes very small, which means that the propagation of the convective wave is degraded in the x-direction. Therefore convergence will be deteriorated. In the incompressible case, the acoustic eigenvalues are formed by the divergence of the velocity field in the continuity equation and by the pressure in the momentum equation. If these components of the inviscid system are discretized impliciteyearly in the y-direction, the determination of the timestep is changed into $\Delta\tau = \Delta\tau_i$. If the flow is aligned to the x-direction, the acoustic cfl_{xa} is equal to 1, and the convective cfl_{xc} will be of the same magnitude if β is chosen appropriately. Then convergence is not deteriorated. The acoustic cfl_{ya} is now bigger than 1 but this is allowed since the acoustic terms are treated impliciteyearly. The numerical anisotropy of the viscous system on the same high aspect ratio grid can also be accounted for, if also the viscous terms are discretized line impliciteyearly in the y-direction. The viscous terms in the x-direction are discretized point-implicit. The discretization of the non-linear convective part of the inviscid system is done expliciteyearly. Velocity upwinding is used and higher order is obtained with the MUSCLE aproach. Due to the preconditioning, this part is stepped with a cfl number in the order of unity. A similar approach is used for low Mach number compressible flows.

The preconditioned semi-implicit line method is used in a multistage scheme because of the stability restrictions on the explicit part. Multigrid is then used as acceleration technique. The convergence is very fast, independent of the aspect ratio and Mach number.

1 Presented by E. Dick

2.3 An additive multigrid method using a filtering concept to solve flow simulations on parallel computers

Mon 11:15–11:45

Luc Fournier
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Since the 80's, multigrid methods are well known to be caracterized by a relatively low parallel efficiency. Nevertheless several works have shown the great interest in these methods for the numerical simulation of industrial flows. Therefore, it seems still useful to develop multigrid methods adapted to parallel computers. Several approaches have already been investigated.

Here, an additive formulation is analysed in detail. This method has been initially proposed by R. Tuminaro [Tum92]. The originality is in the use of filtering techniques in order to disconnect the different parts of the frequency spectrum of the residual. Each part of the filtered residual is used to create a correction problem which is treated on the most adapted grid. After a detailed presentation of the algorithm, a convergence analysis is conducted using properties introduced in [Hac85]. This analysis shows that the additive method should have a convergence rate independent of the mesh size.

This additive multigrid method has been used to solve linear systems involved in compressible fluid flow calculations. The numerical framework is based on the use of unstructured meshes with an agglomeration technique in order to provide coarse grids [LSD92]. We solve the Euler equations for steady flows using a finite volume discretization based on the MUSCL technique. The steady solution is reached using an unsteady formulation with a pseudo-time step. An implicit formulation allows us to define a time step not restricted by a Courant-Friedrichs-Lewy stability condition. So, due to the use of a linearization of the jacobian matrix of the fluxes, each time step results in the resolution of a linear system.

The multigrid method is used to accelerate the resolution of this linear system.

The method has first been validated through a sequential implementation. Then we have developed a parallel version using a mesh partitioning approach. But, as the convergence rate of the additive formulation is worst than the multiplicative one, we decided to use the additive method only for the coarsest levels where the ratio between communication and calculation is unfavorable.

Hac85 *multi-grid methods and applications*, volume 4 of *Springer series in Computationl Mathematics*. Springer Verlag, 1985.

LSD92 M.-H. Lallemand, H. Steve, and A. Dervieux. unstructured multigridding by volume agglomeration: current status. *Computers and Fluids*, 21:3

Tum92 R. S. Tuminaro. a highly parallel multigrid-like method for the solution of the euler equations. *Sci. Stat. Comput.*, 13(1):88–100, January 1992.

1 Presented by Luc Founier

2.4 Multigrid for Multidisciplinary Applications: Strip Steel Production

Mon 11:45–12:15

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Within the GRISSLi project a coupling interface is developed for the coupling of different simulation codes and thus the solution of multidisciplinary problems. The twin-roller production of strip steel states such a multidisciplinary problem combining the flow of the liquid steel (casting) and the mechanical deformation of the solidified steel (rolling). As a near net shape casting method the twin-roller process reduces the production cost of thin strip steel and the manufacturing costs of thin strip steel production plants.

The fluid flow code is coupled with the structure mechanic code via the GRISSLi coupling interface which provides interpolation, communication and management of exchanged data. For fluid flow calculations the L_iSS package developed at GMD is used. L_iSS provides an environment for the parallel multigrid solution of partial differential equations on general domains and is proved to be very efficient for this purpose.

In order to enable a realistic simulation of the steel flow, the Navier-Stokes solver of the L_iSS package had to be extended with a temperature equation, a turbulencemodel, and a solidification model.

Because of the complex flow patterns, the fluid flow simulation for the twin-roller process has to be performed in three spatial dimensions. As a result of this, the L_iSS package has to be extended considerably. On the one hand, this challenging task is simplified by the use of CLIC, communications library for industrial codes. On the other hand, complex numerical questions, such as correct

[1] This work was supported within the GRISSLi-project by the German Federal Ministry for Research and Education, contract no. 01 IS 512 C

treatment of the combination of complex boundary conditions on curved boundaries, have to be solved successfully.

The focus of the talk will be on the L_iSS package as a tool for the parallel multigrid solution of fluid flow problems with respect to the described multidisciplinary application of strip steel production. Several flow simulations and coupled calculation results will be presented.

1 Presented by M.G. Hackenberg

2.5 Robust Algebraic Multigrid Methods in Magnetic Shield Problems

Mon 16:00–16:30

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In this talk we will present a robust and efficient solver for large sparse and poor conditioned linear systems arising from the FE-method for elliptic scalar PDEs of second order.

For a counter example the problem of magnetic shielding is used. Therefore the Maxwell’s equations for stationary objects are reduced to a scalar PDE of second order with appropriate boundary conditions.

In order to solve the equation by means of FEM, a discretization for micro scales is introduced. Especially long thin elements are suggested to keep the number of unknowns small in areas of micro structures.

To achieve an efficient and robust solution strategy the algebraic multigrid method of Ruge and Stüben is used. Additionally three different areas of application are presented for this AMG method, i.e. preconditioner, coarse grid solver for a full multigrid method, and black box solver.

Because this AMG method normally works well for M-matrices, a technique is presented to attain M-matrices, if the underlying linear system arises from an FE-discretization. The method to achieve the M-matrix property is based on the element matrices.

The algorithm was implemented as black box solver in the finite element package FEPP. Therein AMG was applied as preconditioner for the conjugate gradient method.

Some numerical experiments are presented, where long thin quadrilaterals are used with ratio of the longest and shortest side of 1 to 10^{-3}. Additionally parameter jumps of order 10^{-6} to 10^{+6} are considered.

At least in a numerical way, AMG has been proven to be an efficient and robust solver for magnetic shielding problems, if it is used as a preconditioner for the CG-method. In case of long thin quadrilaterals in the discretization the modified preconditioner also behaves very robust.

1 Presented by Stefan Reitzinger

2.6 Multigrid Methods for Higher Order Discretizations of the Navier–Stokes Equations

Mon 16:30–17:00

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For the numerical solution of the Navier-Stokes equations, first order upwind-methods have shown to be very stable discretizations which produce reasonable solutions up to high Reynolds numbers. Moreover, the corresponding algebraic systems have good properties (M-matrix) that allow to construct robust and efficient multigrid solvers. However, the disadvantage is that the discretization error behaves only like $O(h)$ with respect to the mesh size h such that a very fine mesh is required to get high accuracy of the numerical solution. Therefore, one wants to use higher order discretizations. However, for such discretizations the usual multigrid methods do not perform very well. In this talk, some ways to overcome these problems are presented. By means of numerical experiments, the different approaches are evaluated.

2.7 Robust multigrid for edge-oriented discretizations

Mon 17:00–17:30

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Discretization spaces with edge-oriented degrees of freedom, as for instance nonconforming rotated multilinear finite elements, staggered grid discretizations or certain finite volume variants, are often used in practical applications, especially in *Computational Fluid Dynamics*, due to their seemingly excellent numerical properties. Even for complex situations which require highly anisotropic meshes with large *aspect ratios*, i.e. which are needed for resolving boundary layers, there are some theoretical results in the mathematical and engineerical community which address the robustness and efficiency of corresponding multigrid tools.

It is clear that special smoothing operators - ILU, line methods, etc. - are required. However, we have recently figured out the following (surprising) result which is partially contradicting to some of (our own) older statements: *Multigrid for nonconforming finite elements with standard intergrid operators is not stable!*

Additional modifications have to be done in practise for the grid transfer and coarse grid operators. To be precise, we demonstrate that the following ingredients are necessary for robust and efficient multigrid on such highly complex meshes:

- (locally) adaptive prolongation and restriction
- (locally) adaptive construction of coarse mesh operators

Numerical examples show that without these local modifications, multigrid for such discretization spaces may fail! Finally, we demonstrate how these techniques can be incorporated into a self-adaptive framework for various finite element spaces.

2.8 A Modified Full Multigrid Algorithm For Navier-Stokes Equations

Thu 10:45–11:15 b

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A new full multigrid algorithm in which the starting quantities are directly taken from the previous cycle is presented in the paper. This results in no restriction procedure for variables except for residuals. It simplifies the multigrid strategy and the structure of code. In combination with the SIMPLE algorithm, this algorithm is applied to solve fluid flows using collocated grid and higher order schemes for convective fluxes. Since the solution is directly taken from the previous cycle, there is no-matching problem of mass fluxes. The pressure correction equation on the coarse grid is similar to any other variables in this work, different to other works, in which so-called correction of the pressure-correction on the coarse grid is determined.

Accurate solutions are obtained for 2D / 3D complex laminar and turbulent flows such as lid-driven flows in 2D and 3D cavities, flows over backward-facing step, complex turbulent flows over hill, laminar and turbulent flows in curved ducts with strong secondary motion. The modern turbulence models are used for the complex turbulent flows. A speed-up up to about 80 for 2D cases as well as a speed-up up to about 40 for 3D cases have been obtained.

1 Presented by F. Thiele

2.9 Dynamically Adaptive Multigrid on Parallel Computers for a Semi–Implicit Discretization of the Shallow Water Equations

Thu 11:15–11:45 b

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Dynamically adaptive local refinements have the potential to reduce essentially the computational costs of numerical simulations. In meteorological prediction models it would be advantageous to provide high resolutions only where it is necessary (e. g. weather fronts, strong low pressure areas). Calm regions could be calculated with a lower mesh size. Since the weather situation is changing during the simulation, the refinement areas have to be adapted in time. This adaptation has to be performed automatically during the simulation and controlled by a suitable refinement criterion.

For the application meteorology a parallel numerical model with dynamically adaptive local refinements has been developed and implemented. The Shallow Water Equations (SWE) were selected as model equations, which form the dynamical basis of full weather prediction models. On a structured global grid, refinement areas are composed of adjacent rectangular patches, which are aligned to the global grid with refinement ratio $1 : 2$. With a suitable refinement criterion the refinement areas are dynamically adapted to the calculated solution.

As a consequence of the very high resolutions, which become available in adaptive models, the time steps of explicit time schemes become unacceptable small (CLF–criterion). For this reason, a semi–implicit time scheme was developed to increase the lengths of stable time steps. This semi–implicit scheme results in a Helmholtz equation, which has to be solved efficiently on the refined grid in every time step. A multigrid algorithm, which deals with local refinements in a very natural way, is used as solver.

A major effort was spent on designing the numerical model for the usage of parallel computers with distributed memory and to combine adaptivity and parallelism. Grid partitioning of the global grid and of the refinements is performed and the explicit message passing approach was used as parallel programming model.

Asynchronous communication is used to overlap computation and communication as far as possible.

2.10 Multigrid Methods for Low Mach Number Viscous Flows on Adaptive Unstructured Grids

Thu 11:45–12:15 b

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The low–Mach number Navier–Stokes equations seems to be the most adequate model for flows with low (compared to the sound speed) velocities and large temperature variations. These equations provide the results identical to the full compressible Navier–Stokes computations while reducing greatly CPU time.

The advantage of unstructured grids is the relative ease with which complex geometry can be treated. This approach needs the minimum input description of the domain to be discretized and does not tied closely to its topology in contrast to a block-structured grid. The required CPU time to attain the prescribed accuracy may be less than for the block-structure approach due to the much lesser total number of grid cells as a direct sequence of the second advantage of unstructured grids — the easiness of adaptive mesh refinement. Finite volume methods for viscous flow computations are based on body-fitted non-orthogonal grid systems. There are several possibility to discretize equations on structured grid, the extreme ones being: 1) contravariant velocity component on a staggered grid and 2) Cartesian velocity components on a co-located grid. In case of unstructured grid one has to use Cartesian velocities. One can use either semi-staggered grid (the velocity components are stored in the vertices while all other variables - temperature, dynamic pressure, species concentrations - in cells' centers) or a collocated one. The latter seems to be more convenient for multilevel methods and is accepted in the present work.

The aim of the paper is to compare different multigrid approaches to real-life internal viscous flow problems. Among others, algebraic multigrid methods and global solution adaptation coupled to the regular grid refinement are considered.

Examples of computations of flow and deposition in epitaxial reactors used for growth of semiconductor materials are presented.

1 Presented by A. I. Zhmakin

3 Convection Problems

3.1 On the convergence of basic iterative methods for convection-diffusion problems

Wed 9:00–9:45

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We consider linear systems which result from finite element or finite volume discretization of convection-diffusion problems. We analyze the convergence of basic iterative methods of Jacobi and Gauss-Seidel type applied to these linear systems. One known standard result (cf. [1]) for a class of 2D model problems uses the assumption that the underlying triangulation is of weakly acute type (the angles of the triangles are less than or equal to $\frac{\pi}{2}$). The resulting matrix then is an M-matrix and a standard convergence analysis can be applied. In this talk we consider a setting in which the matrix of the discrete problem is not necessarily an M-matrix. In this setting we introduce a few weaker algebraic conditions, e.g. that the matrix is the sum of a symmetric positive definite matrix (diffusion part) and an M-matrix (convection part). Assuming that one or more of these conditions is satisfied we analyze the convergence of basic iterative methods. For a few popular finite element and finite volume methods we show which of these algebraic conditions are satisfied in general.

[1] H.-G. Roos, M. Stynes and L. Tobiska, *Numerical Methods for Singularly Perturbed Differential Equations*, Springer 1996.

1 Presented by A. Reusken

3.2 Application of multigrid methods to 2-D MHD Hartmann flow

Wed 10:45–11:15 b

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In this study, two-dimensional magnetohydrodynamic duct flow problem is solved by using the multigrid method. The problem to be considered is the steady flow of an incompressible electrically conducting fluid in a rectangular duct with uniform applied magnetic field. By solving the governing equation for this flow, a paraboloidal-like velocity profile is obtained. The problem is also solved by Gauss-Seidel and SOR methods beside the multigrid method and it is seen that when the multigrid method is employed, the convergence rate increases dramatically. The multigrid method decreased the error to the order of 10-9 after only 400 iterations and made it oscillatory around the order of 10-12.

1 Presented by Erkan Yeniceri

3.3 On ordering strategies for multigrid methods

Wed 11:15–11:45 b

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While multigrid methods are fast for the iterative solution of ”well behaving” elliptic problems, their convergence rate deteriorates in the case of dominant convection, mainly due to the loss of the smoothing property. Another, similar case of smoothing failure is the occurrence of anisotropy, which is caused either by the equation itself or by the geometric structure (grid/triangulation) which underlies the discretisation.

When one uses finite element methods with piecewise linear functions on a triangulation, the unknowns of the discretisation correspond to the corners of the triangles/tetrahedra.

It is important to notice that in the case of dominant convection the equation also changes its character: from elliptic to hyperbolic. This gives hints for the process of the solution of the arising linear system: methods following the characteristics are appropriate.

Algebraically, this corresponds to a reordering of the unknowns and the application of smoothing methods like Gauß-Seidel smoothing which can be almost an exact solver if applied to upper (lower) triangular matrices While reordering has been carefully studied when dealing with noncircular flow fields, circular flow needs an appropriate block partioning.

If anisotropy occurs, unknowns of spatial dimension may be decoupled, and instead of one 3D problem one is faced with several 2D problems which should be treated separately. In the process of numerical solution, this corresponds to nonoverlapping block partionings. The situation becomes more complicated if the direction of anisotropy varies over the domain.

The idea to find this block partionings is to analyse the structure of the underlying matrix graph and use graph theoretical methods to find unknowns which are strongly coupled and should be treated together.

We propose reordering and block partioning algorithms, which, in connection with suitable (block) iterations, can compensate the smoother’s failure.

3.4 Ordering techniques for convection dominated problems on unstructured three-dimensional grids

Wed 11:45–12:15 b

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For convection-dominated problems in two spatial dimensions, multigrid methods using a simple Gauß-Seidel method as a smoother can lead to excellent results if the unknowns are ordered appropriately.

The difficulties with constructing robust multigrid solvers in three dimensions are much greater than in two dimensions. Many ordering strategies that have been proposed for two-dimensional problems make explicit use of the planarity of the underlying graphs and are not directly applicable to three-dimensional problems.

We will use the graph of a matrix to develop and illustrate ordering techniques for two- as well as for three-dimensional problems. The matrix graph $G = G(A)$ consists of the vertex set $V = V(G)$ and the edges $E = E(G) = \{(i,j) \in V \times V : \ a_{ij} \neq 0\}$. If the matrix A arises from a finite element discretization of a partial differential equation with continuous piecewise linear basis functions in the standard nodal basis, the vertices and edges in the triangulation correspond to the vertices and edges in the matrix graph, resp..

Typically, the size $|a_{ij}|$ of the matrix entries varies considerably over the grid due to the convective term. For the proposed ordering strategies, we neglect small matrix entries and define the graph of dominant entries as the graph with the reduced edge set $E_0 = \{(i,j) \in E : |a_{ij}|$ ‘is large’$\}$.

In three spatial dimensions, we have a structure consisting of vertices and edges, but, additionally, we can involve the faces of the tetrahedra. A sequence of neighbouring faces describes a surface which can be viewed as the counterpart to a (one-dimensional) path in the graph. In turn, an ordering of these surfaces in addition to an ordering within each surface defines an ordering for the vertices in the tetrahedral grid.

The proposed ordering algorithm produces a decomposition of the unknowns into disjoint blocks so that block iterative methods may be applied.

3.5 Adaptive Multigrid Solving of Large Systems of Chemical Reactions with Diffusion and Transport

Thu 14:00–14:30 b

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When doing adaptive simulations, in some cases optimal performance is only obtained when the algebraic data structure is closely connected with the geometry of the underlying grid. In this respect a pointer based matrix graph (such as implemented in the program package UG, see [1]) has several advantages over the classical Harwell-Boeing format. First, the graph structure allows local insertion/deletion of matrix elements in $O(n)$ operations where n is the number of elements to be changed. This can be necessary if moving reaction zones are adaptively resolved. Second, reordering of the unknowns can be easily done, which is necessary to obtain robust multigrid smoothers in regions of dominating convection. Finally, the parallelization could be done on an abstract graph level.

Yet, for systems it is not reasonable to represent every unknown as a node and every matrix entry as a link in the graph, since this introduces a large overhead in both memory and computing time. On the other hand, storing full blocks on the links of the matrix graph is not adequate for large systems of reaction-convection-diffusion equations where the reaction terms usually couple only unknowns at one spatial location with each other. Therefore I introduce a new approach, where I allow the links in the matrix graph to represent sparse matrix blocks. Only the local sparsity pattern is given by an extended Harwell-Boeing type format permitting additionally identification of equal matrix elements.

In my talk, I will demonstrate the efficiency of this approach by several examples. These include 3D simulations of chemical reactions in stirred tanks (joint work with Sven Schmalzriedt, IBVT Stuttgart), as well as 3D simulations of biochemical reactions in saturated porous media (joint work with Christian Wagner, ICA 3, Universität Stuttgart, and Rouslan Nabokov, IWR, Universität Heidelberg).

1 P. Bastian, K. Birken, K. Johannsen, S. Lang, N. Neuss, H. Rentz-Reichert, C. Wieners: *UG - A Flexible Software Toolbox for Solving Partial Differential Equations.* Comput. Visual. Sci. 1, 27–40 (1997)

3.6 An efficient multigrid method for the Morley discretization of the biharmonic equation

Thu 14:30–15:00 b

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Based on a study of convergence proofs for nonconforming multigrid methods, we introduce a new, more effective smoother and prolongation for the Morley discretization of the biharmonic equation, or equivalently, for the $(P_0,$ nonconforming $P_1)$ discretization of the Stokes problem. In our experiments, even the V-cycle with one pre- and post-smoothing turns out to yield uniformly bounded (and small) condition numbers, whereas the costs are equal to such a cycle with standard prolongation and Richardson smoother.

The relation between above discretizations of biharmonic and Stokes problems has been used more often to develop and analyze multi-level methods, in the sense that the analysis took place in the framework of the biharmonic problem. We will follow the opposite approach. Since in contrast to the biharmonic operator, the Stokes operator has full regularity, suitable smoothers developed in the latter framework can be expected to be more effective. In particular, an asymptotic bound $\sim \nu^{-1}$ for the contraction number of the W-cycle can be shown, where ν is the number of smoothing steps, whereas in the framework of the biharmonic equation, a bound $\sim \nu^{-\frac{1}{2}}$ is the best one can hope for. On the other hand, since in the Stokes framework the standard basis is not L^2-stable, to show the smoothing property we had to develop a non-standard smoother involving a conforming multigrid call on a subset of the unknowns.

The standard prolongation used for the Morley finite element space has energy norm larger than 2. This means that the corresponding coarse-grid correction is divergent, and as a consequence, combined with a relatively poor smoother it may even result in a divergent W-cycle. The analysis of additive nonconforming multi-level methods has shown that the energy norms of the iterated prolongations are relevant; (sub-)optimal results can only be expected when these norms are uniformly bounded. However, again for the standard Morley prolongation these norms grow exponentially. Therefore, we introduce a new prolongation based on some local energy minimalization, which at the same time reproduces second order polynomials. In our experiments, the energy norm of this prolongation is less than 2, and the energy norms of the iterated prolongations are uniformly bounded. In our practical algorithm we do not implement this prolongation, but instead we let the prolongation follow by a block Gauss-Seidel step on the sets where we want the energy to be minimized. This results both in a good virtually prolongation and in an effective smoother.

3.7 Construction of Robust Multilevel Splittings

Thu 15:00–15:30 b

Christoph Pflaum
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It will be presented robust multilevel algorithms for anisotropic elliptic equations, a special class of convection diffusion equations, and Stokes’ equation. The multilevel algorithms use semi-coarsening, line-relaxation, prewavelets, and prewavelet like functions. It is proved that the convergence rate of the multilevel cycle is smaller than 0.2 independent of the number of unknowns, the size of the anisotropy, and the size of the convection term. The convection term has to be y-direction (or x-direction). The convergence rate is smaller than 0.2 also in case of strongly varying coefficients in y-direction, some non H^1-elliptic equations and a convection term with a changing convection direction. Furthermore, we prove that the convergence rate of a special multigrid waveform relaxation algorithm is smaller than 0.2 independent of the time interval and the number of unknowns.

3.8 An adaptive smoothing-based multigrid algorithm for flow computations

Thu 16:30–17:00 b

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Numerical simulation of real flow problems often requires significant CPU time even on powerful computers. Therefore accelerating the computations is a critical point in solving such problems.

Accelerating incompressible flow computations with an existing 3D CFD code is discussed here. The single grid code was developed for the artificial compressibility formulation of the incompressible Navier–Stokes equations by the first author (Drikakis et al., 1992,1993), and it was based on a characteristics-based method in conjunction with high-order upwind spatial discretization. The discretization with respect to time was realized by an explicit Runge-Kutta method. Furthermore, a non-linear multigrid algorithm was developed and implemented aiming at accelerating the computations (Drikakis, Iliev, Vassileva 1997,1998). The algorithm combined the full multigrid and full approximation storage (FMG-FAS) schemes. Different prolongation operators were implemented and investigated.

The above mentioned multigrid algorithm was further modified by implementing adaptive smoothing and this modification is the objective of the present talk. By adaptive smoothing we mean that the smoother acts only on an adaptively formed subset ω_S of the grid ω. The choice of the adaptivity criterium is an open question, and it might be problem dependent. We restrict our discussion here to solution of steady problems through an unsteady procedure. Currently we use the following criteria: define $r_P = u_P^{new} - u_P^{old}$, then $\omega_S = \{P : |r_P| \geq \gamma |r_{max}|, P \in \omega\}$. The parameter γ satisfies the condition $0 \leq \gamma \leq 1$. It is obvious, that the subset ω_S is identical with the full grid ω if $\gamma = 0$. In other words, the smoothing is performed adaptively in the subregions where the solution changes rapidly. In addition, we perform a complete smoothing after any ν adaptive smoothings. This is done in order to better propagate the information between different subregions.

The adaptive smoothing can be viewed as a further development of at least two approaches: i) so called ”local solution” method (Drikakis 1994) which is based on reducing the computational domain through the computations; ii) Southwell method for hand–solving systems of linear algebraic equations. The last can be viewed as a variant of Gauss-Seidel method, exploiting adaptive ordering of unknowns, based on the range of residuals.

Results from numerical experiments are presented and discussed in order to demonstrate the impact of the adaptive smoothing on the acceleration of flow computations.

1 Presented by Oleg Iliev

3.9 Smoothers in Coupled Multigrid Methods for the Stokes and Navier–Stokes Equations

Thu 17:00–17:30 b

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Coupled multigrid methods have been proven as efficient solvers for the saddle point problems arising in the discretization and linearization of the incompressible Navier–Stokes equations. In our talk, we consider two classes of smoothers for these methods. On the one hand, these are Vanka–type smoothers where in each smoothing step a number of small linear systems has to be solved. On the other hand, we will study smoothers proposed by Braess and Sarazin, which are based on the solution of a global pressure Schur complement system.

First, we consider the Stokes equations which are discretized with the nonconforming P_1/P_0–finite element method. We will sketched the convergence proof of the coupled multigrid method with Braess–Sarazin–type smoothers. Thus, the results of Braess/Sarazin are extended to nonconforming finite element discretizations. Numerical tests which confirm the theoretical results are presented.

The second part of the talk is devoted to a numerical comparison of Vanka–type and Braess–Sarazin–type smoothers in coupled multigrid methods for the solution of the Navier–Stokes equations. These tests will show the superiority of the Vanka–type smoothers for steady state equations as well as the advantageous behaviour of some Braess–Sarazin–type smoothers for time dependent problems.

1 Presented by Volker John

4 Mechanics

4.1 Quadtree grids and some applications in flow problems

Mon 10:45–11:15 b

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The multigrid method, as is well known, significantly reduces the computational cost of some traditional methods of partial differential equations, since the number of the necessary algebraic operations is proportional only to the first power of the number of unknowns introduced. In order to speed up the method further, this number should be minimized. This can be carried out e.g. by local refinements (either in finite difference or in finite element context). A more efficient technique is the use of the quadtree/octree grids for the discretization procedure. This approach seems to be much more general technique as it can be used not only to produce a nonuniform (but Cartesian) computational grid, but it appears also in quite different fields e.g. in the multipole method. In our talk, we briefly introduce the QT-grid generation algorithm and some multigrid techniques in the QT context. Next, we show some applications to potential problems, shallow water equations as well as multipole, interpolation and boundary element techniques.

4.2 Multigrid methods for stabilized finite elements for elasticity and the application to Prandtl-Reuß-plasticity

Mon 11:15–11:45 b

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We consider multigrid methods for stabilized Q_1/P_0, P_2/P_0 and Q_2/P_1 elements for elasticity, where the hydrostatic pressure is modeled by discontinuous lower order approximations. In the implementation the pressure can be eliminated by static condensation. Then, the resulting system is symmetric and positive definite, and standard multigrid methods can be applied. In addition, we introduce a special construction for the smoother to obtain robustness of the multigrid convergence in the incompressible limit.

The stabilized elements are applied to Prandtl-Reuß-plasticity. The basis for the plasticity computations builds a flexible finite element library which supports adaptive multigrid methods for various discretizations. This is coupled by an abstract interface for the material evaluation at every Gauß-point. The equation in time is discretized by the implicit Euler method, and every time step is solved with a modified Newton method where the defect and the tangent operator is evaluated by a radial return algorithm. We show that the approximation and the multigrid convergence are improved by the stabilization.

The algorithm is realized with the software package *UG*, which is fully supported in parallel. The performance is demonstrated by several examples in 2D with assumed plain strain and in 3D. We investigate the resulting displacements, stresses and hardening parameters after a complete loading cycle, comparing different material laws (perfect plasticity, isotropic linear and exponential hardening) and comparing a full 3D geometry with a reduced 2D geometry.

4.3 Multigrid Methods for Elasto-Plastic Problems with the Stress Concentration

Mon 11:45–12:15 b

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It’s well known that the fracture processes in the solid bodies is determined mostly by the stress distribution near concentrators: micro-holes, crack tips etc. The plastic deformations plays an important role in such processes. Usually in fracture mechanics we solve the problem in the original ”continuous” domain (without the holes and cracks) in order to obtain the local stress field (near concentrators) and then we deal with the elastico-plastic problem for the concentrator in unbounded domain.

We propose a technique of numerical solving the elastico-plastic boundary-value problems for several types of stress concentrators based on multigrid methods. We investigated some varieties of iteration process (such as V- and W-cycles, Richardson’s extrapolation etc). The choice of the most efficient relaxation scheme is also discussed. Comparisons of the computational work of the multigrid methods and the several popular one-grid methods were performed. We also propose a method of the correspondent software development based on the object-oriented approach. We consider C++ class hierarchy that provides a flexible and convenient way of program packages design; C++ class library for the fracture mechanics problems was developed.

4.4 Composite grid solvers for elasticity and plasticity problems

Tue 10:45–11:15 b

Radim Blaheta
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In the finite element analysis of elasticity and plasticity problems, we often require the FE grid which is locally refined to be able to catch a highly nonlinear behaviour of the exact solution in some parts of the problem domain. For this purpose, we can either refine the global FE grid or use composite grids consisting from a global coarse grid and local finer grids (patches). The later approach can be advantageously accompanied by iterative solution methods which include the solution of the global and local subproblems in each iteration.

This contribution is devoted to iterative methods and solvers of this type which are based on the alternating Schwarz algorithm. More precisely, we shall describe the fast adaptive composite grid method (FAC) and similar iterative procedures. We shall present some comparison of these methods and discussion of their convergence and some implementation aspects.

4.5 Robust Multigrid Methods for Parameter Dependent Problems

Tue 11:15–11:45 b

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Several problems in computational mechanics lead to parameter dependent problems of the form

$$\left(A + \frac{1}{\varepsilon}B\right) u = f,$$

with an elliptic operator A, an operator B with non-trivial kernel, and a small, positive parameter ε. We are interested in the construction and analysis of robust multigrid methods for the solution of the arising symmetric and positive definite finite element problem.

Specific examples considered in this talk are nearly incompressible materials and Reissner Mindlin plate models. We shortly present robust non-conforming discretization schemes equivalent to corresponding mixed finite element methods.

We will explain the necessary multigrid components, namely a block smoother covering basis function of the kernel of B, and prolongation operators mapping coarse-grid kernel functions to fine-grid kernel functions. For a large class of problems including Reissner Mindlin models we can prove optimal iteration numbers for a two-level method by an abstract Lemma.

For the example of nearly incompressible materials we present new results for optimal W-cycle convergence rate estimates. Key components are a L_2-like norm depending on the parameter ε, for which the approximation property and smoothing property hold true uniformly in ε. The approximation property involves coarse grid approximation as well as approximation in the grid transfer steps. For the proof of the smoothing property we have to use an interpolation norm between the problem energy norm and the energy norm of the block Jacobi preconditioner. In addition to the obtained results, we will also discuss the missing parts for V-cycle estimates. Numerical experiments indicate optimal rates for the V-cycle, for the problem of nearly incompressible materials and for Reissner Mindlin plate models as well.

4.6 PML - a parallel multilevel system for unstructured grids

Tue 11:45–12:15 b

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Many scientific and engineering problems involve complex boundary value problems for partial differential equations. Especially higher level simulations put up considerable demands on the flexibility, accuracy, and efficiency of the underlying numerical model. Among the most attractive approaches having the potential to meet these requirements are finite element methods combined with adaptive local grid refinement, multigrid solvers, and parallelization techniques.

Within this paper, we present the parallel multilevel system PML. The intention of PML is to provide a light, but efficient interface for implementing parallel, adaptive finite element methods. Based on a distributed multilevel data structure, it supplies methods for the generation and adaption of local multigrids consisting of tetrahedral cells. Curved boundary surfaces are approximated by an unique rational point normal interpolation. To optimize performance on parallel computer systems, PML offers predictive load balancing using either METIS or CHACO for mesh partitioning. A special feature of PML is the support for periodic boundaries. This is of particular importance for our primary target application – the large eddy simulation of complex turbulent flows.

A major part of the paper is devoted to the basic design principles of PML. Further, the scalability of parallel mesh adaption is analysed. This is done theoretically as well as experimentally, using the visualization tool VAMPIR. Finally, we discuss our experiences with first applications including diffusion problems, and – as far as completed – compressible flows.

1 Presented by Jörg Stiller

4.7 Recent developments in Schwarz algorithms for the h-p-version of the finite element method

Thu 9:00–9:45

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We consider the Dirichlet-Dirichlet Domain Decomposition (DD) algorithms for systems of algebraic equations of h-p-version of the finite element method for second order elliptic equations. We split the problem of the DD preconditioning into the subproblems of preconditioning of the internal problems on the subdomains of decomposition, of Schur complement and of the discrete harmonic prolongations of polynomials from the interface boundary inside subdomains of decomposition. In the case of the square reference element equipped with the tensor product polynomial space, we are able to suggest efficient and cheap preconditioners for every mentioned component. As a result, we come to the DD preconditioner which is spectrally equivalent to the global stiffness matrix and requires $\mathcal{O}(h^{-2}p^3)$ operations for solving the system with such preconditioner for the matrix. The obtained finite-difference-like preconditioner for internal problems makes it possible further improvement of algorithm on the basis of known efficient solution techniques. Most of the results except for preconditioning of internal problems are expandable to the case of triangular elements. The algorithms are highly parallelizable. Some results of the report have been obtained jointly with S. Jensen and with J. Fish and J. Flaherty.

5 Homogenization

5.1 Coarse grid spaces for domains with a complicated boundary

Mon 14:00–14:30

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Elliptic boundary value problems are often posed on complicated domains which cannot be covered by a simple coarse initial grid as it is needed for classical multigrid–like iterative methods where the coarse grid equations are solved exactly. Several solutions to this problem are presented for the case of homogeneous Dirichlet boundary conditions. The first technique is to construct appropriate subspace decompositions by way of an embedding of the domain under consideration into a square or a cube. The second technique is even simpler. It is shown that the condition number of finite element discretization matrices remains uniformly bounded independent of the size of the boundary elements provided that the size of the elements increases with their distance to the boundary. This fact allows the construction of simple multigrid methods of optimal complexity for domains of nearly arbitrary shape.

5.2 Multi-grid methods for PDEs on complicated domains

Mon 14:30–15:00

Stefan Sauter
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In our talk we present composite finite elements for the solution of PDEs on complicated domains. Efficient solvers as multi-grid methods are based on a multi-scale discretization of the problem. However, for complicated domains and/or for problems with highly oscillating coefficients, coarse-scale discretizations are not available by using standard finite elements. The minimal dimension of classical finite element spaces is proportional to the number and size of geometric details. Composite finite elements overcome this difficulty by relaxing the condition “a finite element mesh has to resolve the boundary” by the condition “the domain has to be covered by the finite element mesh”. Hence, the minimal dimension of composite finite element spaces is independent of the number and size of geometric details. The geometry is incorporated in the finite element functions in an appropriate way. The approximation property of composite finite elements can be proved in the same generality as for classical finite elements.

In our talk, we extend composite finite elements to problems with jumping coefficients and present an algebraic variant of the multi-grid algorithm based on composite finite elements: For a given fine-grid mesh and given fine-grid equation a hierarchy of coarse scale problems is set up along with appropriate prolongation operators in a black-box fashion so that the fine grid equations can be solved via a multi-grid algorithm. We illustrate the efficiency of the method by numerical experiments for the (indefinite) Helmholtz-problem.

5.3 A New Algorithmic Approach to Multigrid-Homogenization

Mon 15:00–15:30

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Elliptic boundary value problems with diffusion coefficients that rapidly and strongly vary on fine scales pose severe problems to standard multilevel methods. This is because those methods construct coarse grid operators which do not properly approximate the coarse scale structure of the problem. For instance, even if the fine scale problem is isotropic the coarse scale operator can be anisotropic.

In analogy to the asymptotic case, the construction of well-chosen coarse grid operators is called *numerical homogenization*. Recently, several researchers discovered that, while standard multilevel methods suffer, the multiscale structure itself is ideally suited for accomplishing that task. In principle, three approaches have been suggested:

- Methods based on successive Schur-complements, computable using wavelet bases. However, in this approach the coarse grid operator becomes global and one does not obtain information about effective diffusion matrices.

- Methods based on ideas stemming from algebraic multigrid, like matrix-dependent prolongations. These methods can also be viewed as a kind of ILU-localization of the (global) Schur-complement approach. They work correctly for layered materials and certain probability densities. However, there are problems like checkerboard-structures where these methods do nothing better than standard multigrid.

- Semi-analytical methods based on asymptotic results. These methods work correctly, however, only for quite a restricted class of problems: diffusion coefficients stemming from periodic media.

By analyzing all these drawbacks the author has realized the importance of conserving from fine to coarse scale those properties which are continuous with respect to the so-called H-convergence of diffusion matrices. This analysis has led to a new, *solution-dependent* way of constructing coarse grid operators, iteratively within a multigrid cycle. Numerical examples will show the promising features of the new approach.

6 Wavelets

6.1 On the Treatment of Bounded Domains and Boundary Conditions in Adaptive Wavelet Methods

Mon 14:30–15:00 b

Angela Kunoth
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For the numerical solution of elliptic partial differential equations, embedding the domain into a larger simple one, called fictitious domain, has been a favorable method, in particular, for domains with complicated boundaries. This approach in combination with the idea of explicitly treating essential boundary conditions by means of Lagrange multipliers decouples the differential operator from the boundary conditions as much as possible. In a weak saddle point formulation of the problem, stable discretizations on the domain and the boundary are then enforced by the corresponding Ladyšenskaja–Babuška–Brezzi (LBB) condition.

In my talk, I would like to present some new ideas how to use an adaptive wavelet-based method for this saddle point problem combined with a strategy how to satisfy the LBB condition for the problem at hand.

6.2 Construction of Globally Continuous Biorthogonal Wavelet Bases on Domains in $I\!R^2$

Mon 15:00–15:30 b

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In order to solve partial differential equations or boundary integral equations with a conforming Wavelet-Galerkin-Scheme, globally continuous biorthogonal wavelet bases are required with the following properties

- norm equivalences in a certain range of Sobolev spaces,
- vanishing moments,
- approximation order,
- boundary value conditions.

In this talk we present a construction that utilizes a domain decomposition strategy. A biorthogonal wavelet system is construcuted where the biorthogonality is given with respect to a modified scalar product. These basis functions are shown satisfy the properties mentioned above.

6.3 Piecewise Linear Prewavelets on Arbitrary Triangulations

Mon 16:00–16:30 b

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Arbitrary triangulations are of great interest in a variety of application areas such as the treatment of partial differential equations or the description of large data sets, e.g. in terrain modelling or computer graphics. Finding piecewise linear prewavelets of small support for sequences of uniformly refined triangulations has thus been a goal under different circumstances (Kotyczka & Oswald, Stevenson for PDEs, Floater & Quak for terrain modelling). In this talk, the authors’ results concerning smallest support, stability, linear independence and efficient filterbank algorithms are presented and compared with related ones by Stevenson.

1 Presented by Ewald G. Quak

6.4 Restricted Nonlinear Approximation and Applications

Mon 17:00–17:30 b

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Nonlinear approximation describes to some extent an ideal state in assuming for example the knowledge of all infinitly many wavelet coefficients of a function. Therefore one cannot hope to achieve proved possible convergence orders in a strong sense by any realistic adaptive numerical method. Hence one has to weaken this ideal state somehow. This was one of the starting points for a systematic study of restricted nonlinear approximation in a recent paper by Cohen, DeVore and Hochmuth. Their analysis provides complete characterizations in terms of scales of Besov spaces.

In this talk we explain how restricted nonlinear approximation covers in particular numerical discretizations with respect to graded meshes adapted to apriori known singularities in the solutions of boundary integral equations. Moreover we describe how restricted nonlinear approximation takes place within the poles uniform approximation and nonlinear approximation. Finally we discuss (besides others) various kinds of tree-type and tresholding algorithms on this background.

[2]The work of the author has been supported by the Deutsche Forschungsgemeinschaft (DFG) under Grant Ho 1846/1-11.

6.5 Adaptive Multiscale Methods for Saddlepoint Problems

Mon 16:30–17:00 b

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Recently, Dahlke, Dahmen, Hochmuth and Schneider introduced a-posteriori error estimates for Galerkin schemes for elliptic operator equations using stable multiscale bases. Based on this, they were able to construct adaptive space refinement strategies that could be proven to guarantee convergence.

Since their construction relies on positive definite operators, saddle point problems are ruled out. In this talk, we present some ideas and results for constructing convergent adaptive multiscale methods for saddle point problems. Moreover, we give a general criteria for ensuring the *Ladyshenskaja-Babushka-Brezzi*–condition for adaptively refined multiscale space. The general theory will be specified for a suitable multiscale wavelet discretization for the Stokes problem introduced by Dahmen, Kunoth and Urban.

1 Presented by Karsten Urban

7 Physics

7.1 From Molecular Dynamics to Continuum Models: A Numerical Approach

Tue 10:45–11:15

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This talk will describe a numerical approach for the derivation of macroscopic equations (partial differential equations), from simulations of molecular models. Its demonstration will be given for examples starting from the simple Brownian motion to more complex fluid models such as the hard sphere (HS) and Lennard-Jones (LJ) models. The resulting fluid dynamics equations and their deviation from Navier-Stokes equations, especially in the rarefied gas regime, will be discussed. The main component of the method are coloring schemes which track properties such as mass momentum and energy and their dynamics for a collection of particles over proper space and time scales, and deducing their dynamics on the larger scales by regression analysis. These schemes allow the construction of a hierarchy of models describing the phenomenon at different space-time scales. It can be used also for accelerating molecular dynamics computation by exploiting their multiscale structure.

1 Presented by Shlomo Ta’asan

7.2 From Stochastic Lattice Dynamics Models to Partial Differential Equations

Tue 11:15–11:45

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In this talk we consider stochastic lattice dynamics models, such as the Ising model, and we derive partial differential equations that describe the behavior of these models on large (macroscopic) scales in terms of spins densities. This is our first step in constructing space-time multiscale techniques for the analysis and the efficient simulation of lattice models. Our numerical approach is based on using coloring schemes that track the spins propagation, and it leads to derivation of finite difference equations describing the dynamics of the spins density on large space-time scales. The methods have been applied to Kawasaki model where a non-linear diffusion processes governs the densities dynamics. Our numerical results agree with the expected theoretical ones.

A related free boundary problem is considered as well. It describes the large scale dynamics of the macroscopic interface defined between large clusters of spins of different signs.

The method seems to be general and can be used in the study of macroscopic behavior for a variety of stochastic dynamics models.

1 Presented by Ira Livshits

7.3 Error analysis for a potential problem on locally refined grids

Tue 11:45–12:15

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A possibility for the simulation of a biomolecular system in an aqueous solvent is to use a continuum model for the solvent. The evaluation of so-called solvation energy coming from the electrostatic interaction between the solute and the water molecules surrounding it is important for a Monte Carlo simulation with this model. In these simulations, we have to deal with a potential problem with jumping coefficient and with a known boundary condition at infinity. One of the advanced ways to solve the problem is to use a multigrid method on locally refined grids around the solute molecule. In this paper, we focus on the error analysis of the numerical solution and the numerical solvation energy obtained for the locally refined grids. Based on a rigorous error analysis via discrete approximation of the Green function, we show the proper way to construct a composite grid, to discretize the discontinuity in the diffusion coefficient and to interpolate the solutions at the interface between the fine and coarse grids. The error analysis developed is confirmed by the numerical experimental results.

1 Presented by Takumi Washio

8 Helmholtz

8.1 Multigrid Wave-Ray Algorithms for Helmholtz Equations

Wed 14:30–15:00

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In this work we present some applications of the multigrid wave-ray algorithm (Brandt, Livshits). Initially, they were developed to solve Helmholtz equations with radiation boundary conditions. The accuracy of approximate solutions for these problems, defined on the infinite domains, is discussed.

We also consider some modifications of the algorithms that makes them applicable for solving Helmholtz equations with different boundary settings.

1 Presented by Ira Livshits

8.2 Hierarchical Bases for the Indefinite Helmholtz Equation

Wed 15:00–15:30

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For the indefinite Helmholtz equation, straight-forward multigrid solvers lose their efficiency and show slow convergence or even divergence for high wave numbers. An equally insufficient performance can be observed with a standard hierarchical basis approach.

The reason for this is that not all of the error frequencies can be treated by standard multigrid, especially those frequencies which are solutions of the homogenous equation can be totally invisible for the standard solvers because of their small residuals. Brandt and Livshits suggested an approach by introducing so-called ray cycles into their multigrid scheme which considered the irremovable errors as a superposition of plane waves. We adopt this approach by using a special hierarchical basis in which the piecewise linear basis functions are multiplied by wave functions with the appropiate wave length.

1 Presented by Michael Bader

9 Maxwell

9.1 Multigrid FEMs in Clinical Cancer Therapy Planning

Tue 12:15–12:45

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The author describes the role of various multigrid methods in the context of a recent planning system for the cancer therapy hyperthermia. Within the rather sophisticated integrated software environment HyperPlan, three types of PDEs have to be solved to high efficiency and medical reliability: (I) high frequency Maxwell’s equations for the total system individual cancer patient, water bolus, radiofrequency applicator, air, (II) the linear Bioheat-Transfer Equation (BHT) that models the distribution of heat in the human body, and (III) nonlinear extensions of the BHT equation, which include systemic physiological effects of the patient body. For the indefinite functional in case (I), a multiplicative MG method has been designed (in cooperation of the Berlin and the Augsburg multigrid groups), which involves a hybrid smoother to take special care of the unwanted nullspace of the curl-operator; even though numerical experiments confirm the typical multigrid complexity, the number of MG cycles needed is regarded as still too high. As for the purely elliptic case (II), additive multigrid methods (KASKADE with BPX preconditioner) are applied; recently, a possible coupling of domain decomposition methods with subdomain CCG methods has been studied by Lipnikov and the author, which are presently under further investigation. As for the nonlinear models (III), two approaches are followed: (a) a recently improved adaptive Rothe method (due to Lang) is used to solve the time dependent problem up to the stationarity, (b) the recently suggested global adaptive multigrid solver for nonlinear elliptic problems (NEWTON-KASKADE by the author and Weiser) is directly applied to the stationary problem. In the latter case, a slight difficulty arises, since the underlying functional is not globally convex, but only decent in a neighborhood of the solution. Finally, optimal temperature distributions for different virtual patients (corresponding to anonymized real cancer patient data) are visualized.

9.2 Multigrid in $H(\mathrm{div})$ and $H(\mathbf{curl})$

Wed 16:00–16:30

Ralf Hiptmair
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Variational problems arising from the inner products in the spaces $H(\mathrm{div}, \Omega)$ and $H(\mathbf{curl}, \Omega)$ typically lack the kind of ellipticity that makes plain multigrid work in the case of second order elliptic problems. The fault lies with the large kernels of the corresponding differential operators. They contain numerous highly oscillatory eigenfunctions that, however, fail to be associated with large eigenvalues. Conventional local smoothing is doomed, thus.

A remedy is offered by Helmholtz–decompositions, orthogonal splittings of the spaces into the kernel of the differential operator and its orthogonal complement. It turns out that the smoother remains effective with respect to error components in the latter. The former can be tackled based on a representation through potentials.

What renders this idea computationally feasible is the existence of discrete potentials in a finite element setting, provided that the appropriate finite element schemes are used. Those are Raviart–Thomas elements for $H(\mathrm{div})$ and Nédélec’s elements for $H(\mathbf{curl}, \Omega)$.

Incorporating smoothing in potential space yields a multigrid method whose performance matches the usual multigrid efficiency for 2nd order problems. This talk will discuss approaches to the theoretical analysis of the scheme.

9.3 Multigrid Methods for Magnetic Field Problems

Wed 16:30–17:00

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The coupling of Finite Element Methods (FEM) and Boundary Element Methods (BEM) is being applied to magnetic field problems since complex geometries or unbounded domains have to be considered. Moreover, if moving geometries are present, BEM can simplify calculations since locally no volume meshes are required. The coupling is being realized efficiently by Domain Decomposition (DD) methods allowing easily the application of parallel algorithms. Besides DD a second concept of parallelization (local parallelization) is being used in order to maintain parallel efficiency in the case of fixed, non-reducible BE–domains.

In our algorithm, multigrid methods are used as preconditioners for several types of subproblems arising from FEM or BEM within the DD framework. In particular operators with different mapping properties of integrating and differentiating type occur. It turns out that concepts originating from BEM are well suited for FEM and vice versa.

While 2D magnetic field problems lead to scalar equations, 3D problems involve usually a vector potential as primary unknown. Because of the infinite dimensional kernel of the differential operator, gauging conditions are imposed in order to guarantee unique solvability. We propose and analyze several types of gauging conditions. Moreover, we introduce the coupling of vector-valued FEM (vector potential) and scalar BEM (scalar potential) based on non-overlapping DD methods similar to the 2D case.

[3] Acknowledgments: This work has been supported by the Austrian Science Fund – ’Fonds zur Förderung der wissenschaftlichen Forschung’ – within SFB 013.

9.4 Multigrid method for a global conductor in the Earth's ionosphere

Wed 17:00–17:30

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The mathematical model of an electric conductor is a boundary value problem for an elliptical equation. It's coefficients form a nonsymmetrical tensor because of Hall effect. We replaced traditional statement with new one in which the operator is symmetrical and positive definite.

Finite element equations are designed as the conditions of a minimum of the energy functional. The matrix of the system of linear algebraic equations is symmetrical and positive definite. It's condition number has the same dependence of grid step as one for the Poisson equation. We use a multigrid method to obtain the solution of the system.

As far as the Earth's ionosphere is concerned the main difficulty is due to huge values of coefficients in a thin strip near the boundary. It represents so called equatorial jet. We separate this singularity by a special boundary condition. It's approximation gives a subsystem with 5-diagonals symmetrical matrix in the system of finite element equations. The estimation of the approximation error for the boundary condition is of the same order as one for the equations inside the domain. In multigrid method we use special interpolation formulas for parameters at the boundary to keep the same structure of equations for all grids. The convergence rate of multigrid iterations is approximately the same as ones for boundary value problems with insulator or superconductor at the boundary.

We conducted numerical experiments with the designed model on the base of available data of satellite and ground based measurements. As a result the models of electric fields and currents distributions in the Earth's ionosphere are designed for substorms and for quiet geomagnetic conditions.

The designed mathematical model of the ionosphere as a global conductor is also using in our more general models of the Earth's magnetosphere. In these models the ionosphere is a passive object in that electric energy of magnetospheric generators dissipates.

10 Nonlinear Problems

10.1 Some Local/Parallel Algorithms for Nonlinear Elliptic Equations

Tue 9:45–10:15

Jinchao Xu
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In this talk, some local and parallel adaptive finite element algorithms for nonlinear elliptic equations in both two and three dimensions will be reported. These algorithms show that, for a solution to some nonlinear elliptic problem, low frequency components can be approximated well by a standard finite element discretization on a relatively coarse grid and high frequency components can be obtained by some linearized discretization on some local fine grid in some parallel procedure. The theoretical tools for analyzing these algorithms are some local a priori and a posteriori error estimates for finite element soultions on general shape-regular grids. Multigrid and domain decomposition techniques both play important roles in this approach.

1 Presented by Jinchao Xu

10.2 On monotone iterations for variational inequalities

Wed 14:30–15:00 b

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A wide range of free boundary problems occurring in engineering and industry can be rewritten as a minimization problem for a strictly convex, piecewise smooth but non–differentiable energy functional, or even more, as a variational inequality. The algebraic solution of the related discretized problem is a very delicate question, because usual Newton techniques cannot be applied.

We propose a new approach based on convex minimization and constrained Newton type linearization. While convex minimization provides global convergence of the overall iteration, the subsequent constrained Newton type linearization is intended to accelerate the convergence speed. We present a general convergence theory and discuss several applications

10.3 Cascade iterative algorithms in finite element method for non-linear elliptic equations

Wed 15:00–15:30 b

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Dirichlet problem is considered for a weakly nonlinear elliptic equation of second order. Implementation of standard finite-element method with piecewise linear elements on triangles results in the system of nonlinear algebraic equations. To solve it, a cascade organization of conjugate-gradient method is used on a sequence of nested grids and gives a simple version of multigrid without preconditioning and projection onto the coarser grid. Cascade algorithm begins at comparatively coarse grid, where the number of unknowns in discrete nonlinear system is less of several orders than this number at finest grid. Therefore we suppose that this system is previously solved with sufficiently high accuracy and appropriate computational complexity. At each finer grid, the nonlinear system is linearized by Newton method with ”frozen derivative” and is approximately solved by conjugate-gradient method; initial guess is obtained by interpolation of approximate solution from the previous coarser grid. It is proved that this cascade algorithm has the same optimal exponent in computational complexity as the traditional multigrid, when they reduce iteration error to the level of discretization one in energy norm. Some numerical experiments confirm this theoretical result.

10.4 Nonlinear space decomposition for degenerated and singular nonlinear equations and some asyncronous versions

Thu 14:00–14:30

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Domain decomposition (DD) and multigrid (MG) have been intensively studied for linear elliptic problems. Their convergence can be analysed using the framework of space decomposition and subspace correction. Using this framework, we extend the DD and MG methods to a class of nonlinear problems. We shall first show that our proposed algorithms have a convergence rate which is as good as for linear elliptic problems when the nonlinear operator is strongly monotone and Lipschtz continuous. We shall also give a convergence rate estimate for nonlinear degerated and singular elliptic problems. Applications will be discussed for nonlinear p-Laplace equation and the full potential flow equation. Some numerical results will be reported. Convergence of some asynchronous version will also be discussed.

1 Presented by X.-C. Tai

[4]This work was partially supported by the Norwegian Research Council Strategic Institute Programm within Inverse Problems at RF–Rogaland Reseach and by Project SEP-115837/431 at Mathematics Institute, University of Bergen.

[5]This work was partially supported by NSF DMS-9706949 through Penn State.

10.5 Solving Optimal Control Problems by Multigrid Methods with Transforming Smoothers

Thu 14:30–15:00

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Optimal control problems in PDE are frequently encountered in inverse modeling, shape optimization or process control problems. Typically these problems evolve from simulation tasks defining some output states to be influenced by some input controls. Recent efficient numerical methods typically rely on the *direct discretization approach*, which treat the states and controls together as unknowns of a discretized finite dimensional constrained optimization problem and apply iterative methods of sequential quadratic programming (SQP) type. Despite the resulting large number of variables this approach has proven very successful since it enables a simultaneous solution of the optimization and the simulation problem.

At the core of all SQP type methods lie *Karush-Kuhn-Tucker (KKT)* systems. They can be considered special saddlepoint problems which, however, differ from the ones from Stokes or Navier-Stokes discretizations. In this talk a novel numerical multigrid approach is presented to the numerical solution of such KKT systems. This approach is based on iterative null-space methods employing so-called *transforming smoothers*.

A multigrid convergence proof for a model problem as well as numerical results for a practical inverse modeling problem from groundwater flow are given.

10.6 Least-Squares Mixed Finite Elements for a Nonlinear Elliptic Problem: A Gauss-Newton Multilevel Method

Thu 15:00–15:30

Gerhard Starke
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Least-squares finite element methods for first-order system formulations of linear second-order elliptic boundary value problems have received much attention in recent years. In this talk, this methodology is applied to a nonlinear problem arising from an implicit time discretization of variably saturated subsurface flow. This approach simultaneously constructs approximations to the flux in Raviart-Thomas spaces and to the hydraulic potential by standard H^1-conforming linear finite elements. Two important properties of the least-squares approach which will be utilized are:
(i) the local evaluation of the (nonlinear) least-squares functional serves as an a posteriori error estimator, and
(ii) the Gauss-Newton method provides a robust iterative solution method for the resulting nonlinear least-squares problems.

The first of these properties leads to adaptive refinement strategies based on the local least-squares functional which allow the resolution of the steep saturation fronts which occur as water infiltrates dry soil. The focus of this presentation is on the second property, in particular, on an inexact version of the Gauß-Newton method which combines robustness with efficiency.

Adaptive multilevel methods are used for the linear least-squares problems arising in each step of the Gauß-Newton method. For the Raviart-Thomas spaces this requires an adaptation of the multilevel method by Arnold, Falk and Winther to locally refined triangulations. The crucial part of the Gauss-Newton multilevel method is the proper choice of the stopping criterion for the inner (multilevel) iteration based on the error of the outer (Gauss-Newton) iteration. In the context of nonlinear least-squares finite element techniques, this accuracy matching may be based on the fact that the least-squares functional itself serves as an error measure. Computational experiments conducted for a realistic water table recharge problem illustrate the effectiveness of this approach.

11 Inverse Problems

11.1 A Multilevel approach for the retrieval of atmospheric trace gases

Wed 16:00–16:30 b

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Remote sounding from earth satellites using Fourier spectroscopy has become the main tool to monitor global changes in the atmosphere, like the formation of the ozone hole or the green house effect. The limb sounding observation technique consists of the measurement of emission spectra for different scan angles from the satellite through the atmosphere. By a least-squares fit procedure, temperature profiles and density distribution profiles of the major trace gases, are retrieved. The fit procedure is based on a forward model for relation between the unknown temperature and density distribution profiles and the measured emission spectra. Mathematically, the forward model is given by the radiative transfer equation, a nonlinear Fredholm integral equation of the first kind. Therefore, for the inversion of the radiative transfer equation a non-linear ill-posed problem has to be solved.

Due to the noise sensitivity of the inverse problem, regularization techniques that incorporate additional information, have to be applied in order to compute meaningful solutions. For Tikhonov regularization, which aims at smooth solutions, an additional term consisting of a Sobolev norm of certain degree of the solution is added to the least-squares functional. On the other hand, the optimal estimation technique intends to obtain a solution close to a known a priori profile. As additional constraint the difference between the solution and the a priori profile measured in the L_2 norm, weighted by the variance-covariance matrix of the a priori information, is added to the least-squares functional.

For the minimization of the resulting functional a nonlinear multigrid method in the form of the full approximation scheme (FAS) is applied. The coarse grid problems are formed by restricting the regularized least-squares functional to coarse grid subspaces. As error smoother the so called onion-peeling method, a nonlinear Gauss relaxation starting at the uppermost altitude grid point downwards to the lowest altitudes point, is used.

The performance (number of forward calculations) of the multigrid method is compared with standard retrieval methods (e.g. Levenberg-Marquardt iteration).

11.2 A Multigrid Approach For Minimizing A Nonlinear Functional For Digital Image Matching

Wed 16:30–17:00 b

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We consider an application of multilevel methods to medical imaging. The specific problem arises in the processing of images of the human brain. Two different $3D$ grey scale images have to be matched: the first one is a histological, high resolution image with distortions called the template $T(x)$. The other one a MRT image called the reference $R(x)$ which provides only macroscopical resolution, but the true geometry. One looks for a displacement field u which yields optimally matched images. The difference of the two images is measured by their L_2 difference:

$$D(u) = \int_\Omega (T(x - u(x)) - R(x))^2 d\Omega,$$

and should be minimized. This yields an ill conditioned inverse problem for the displacement field u. Due to physical considerations, a regularizing term proportional to the elastic energy $a(u, u)$ of the deformation is added. Therefore a functional of the form

$$J(u) = a(u, u) + D(u)$$

has to be minimized. The corresponding Euler equations are a nonlinear coupled system of boundary value problems. It is discretized by finite differences. We describe numerical results for different treatments of the nonlinearity (e.g. steepest descent methods with $1D$ minimization, FAS and different relaxation methods) coupled with a multigrid method.

Since the functional may have many local minima a multilevel nested iteration approach is essential to find suitable - hopefully global - minima. In this case the images were be transformed on a coarser resolution. On this resolution only the

low frequencies of the images were presented. Now we start the minimization process on this resolution and interpolate the resulting displacement fields on the next finer resolution level, where the minimization process is performed again. This procedure is repeated until the finest level is reached. Now we have a suitable starting point on the finest resolution and the minimization process can be finished.
This approach has resulted in a substantial reduction of computing times and has made possible to treat the $3D$ problem. Further reduction of the processing time is possible by parallelization. We describe results for an implementation using KeLP as a basic tool.

1 Presented by Stefan Henn

11.3 Multigrid Methods for an Inverse Potential Problem

Wed 17:00–17:30 b

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In electrocardiographic diagnostic applications, inverse problems of the following form occur. The electric field is described by a potential equation with spatially varying dielectric constants. Natural boundary conditions are prescribed for the outer boundary of the domain. Additionally, measurements provide values of the potential on the boundary corresponding to extra Dirichlet data. Thus, in a forward problem, the boundary is over-specified. In the inverse problem, however, the right hand side (that is the source term for the potential) is unknown in parts of the domain. Alternatively the problem may have an interior boundary, where no boundary data are given.

This inverse problem is ill-posed and its treatment requires careful use of regularization techniques. The corresponding discrete systems are badly conditioned, even after regularization. Thus the conventional iterative solvers converge very slowly and the current computational solutions are all extremely compute intensive. Clearly, it would be an important progress if multigrid could be used to speed up the solution process.

As a prototype for the general problem we study the initial value problem for Laplace's equation. In simple geometries, this model problem can be analyzed using Fourier techniques and thus it is a first starting point for exploring various alternatives how to incorporate multigrid. Besides the use of multigrid in repeated solutions of the forward problem, as they occur in standard iterative processes for the inverse problem, we will also discuss the possibility to solve the inverse problem directly with a multigrid approach.

1 Presented by Marcus Mohr

12 Implementation Aspects

12.1 Cache Based Multigrid on Quasi-Structured and Unstructured Grids

Thu 10:45–11:15

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Computers today rely heavily on good utilization of their cache memory subsystems. Compilers are being optimized for business applications, not scientific computing ones, however. Automatic tiling of basic numerical algorithms involving sparse matrices and non-uniform grids is simply not provided by any compiler. Thus, absolutely terrible cache performance is normal for scientific computing applications.

Multigrid algorithms combine several numerical algorithms into a more complicated algorithm. Algorithms are derived that allow for data to pass through cache exactly once per multigrid level during a multigrid cycle before the level changes. This is optimal cache usage for large problems that do not fit entirely in cache.

The new algorithms are designed to provide the exact same solution (bitwise compatibility) as an equivalent standard multigrid algorithm using the same ordering for the iterative solver. Hence, existing multigrid theory can be used to ensure both convergence and the rate of convergence.

Non-uniform meshes and variable coefficient partial differential equations make designing cache based multgrid algorithms sigificantly different from cache based multigrid algorithms for similar problems on uniform or tensor product meshes.

Quasi-uniform and unstructured grids have similar, but different properties that must be addressed separately.

The new algorithms run faster than the standard multigrid implementations. Finally, the new algorithms are designed to run well on any cache based computer and are not tailored to any particular computer system.

1 Presented by Craig C. Douglas

12.2 Cache-aware Multigrid for 3D Elliptic Equations

Thu 11:15–11:45

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Conventional implementations of iterative numerical algorithms, especially multigrid methods, merely reach a disappointing small percentage of the theoretically available CPU performance when applied to representative large problems. One of the most important reasons for this phenomenon is that the need for data locality due to poor main memory latency and limited bandwidth is entirely neglected by many developers designing numerical software. Only when most of the data to be accessed during the computation are found in the system cache (or in one of the caches if the machine architecture comprises a cache hierarchy) fast program execution can be expected. Otherwise, i.e. in case of a significant rate of cache misses, the processor must stay idle until the necessary operands are fetched from main memory, whose cycle time is in general extremely large compared to the time needed to execute a floating point instruction. In this paper, we extend techniques developed to improve the cache performance of two-dimensional multigrid algorithms for the three-dimensional case. We will introduce sophisticated blocking techniques and program restructurations that are significantly more involved than in the two-dimensional case. Numerical experiments are presented showing the efficiency of our cache-aware methods.

Presented by Markus Kowarschik

12.3 Key Based Multigrid on Adaptive Grids

Thu 11:45–12:15

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We report on an adaptive, parallel, finite difference, additive multigrid code for the solution of partial differential equations. The code is based on two concepts: We use Hilbert’s space-filling curves for the key based addressing of vertices and geometric entities. This enumeration scheme leads to a cheap and efficient way of geometric, dynamic data partitioning and migration on a parallel computer. Furthermore, it is used for the hash storage of the vertices, which is the novel second concept. This way traditional tree data structures are avoided, which can be annoying especially in parallel implementations.

1 Presented by Gerhard Zumbusch

13 Discretizations

13.1 Multigrid for the mortar finite element method

Thu 16:00–16:30

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A multigrid technique for uniformly preconditioning linear systems arising from a mortar finite element discretization of second order elliptic boundary value problems will described. These problems are posed on domains partitioned into subdomains, each of which is independently triangulated in a multilevel fashion. The multilevel mortar finite element spaces based on such triangulations (which need not align across subdomain interfaces) are in general not nested. Suitable grid transfer operators and smoothers are developed which lead to a variable V-cycle preconditioner resulting in a uniformly preconditioned algebraic systems. Computational results illustrating the theory will also presented.

13.2 Error Analysis for Sparse-Grid Recombination

Thu 16:30–17:00

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Sparse grids were introduced in 1990 by Zenger [1], in order to significantly reduce the number of degrees of freedom that describe the solution to a discretized partial differential equation, while causing only a marginal increase in representation error relative to the standard discretization. Representing a solution as a piecewise d-linear function on a conventional d-dimensional grid of mesh width h requires $\mathcal{O}(h^{-d})$ degrees of freedom, while the representation error is $\mathcal{O}(h^2)$. The representation as a linear combination of hierarchical basis functions on a sparse grid requires only $\mathcal{O}(h^{-1}(\log h^{-1})^{d-1})$ degrees of freedom. In fact, this is only a one-dimensional complexity, while the approximation error is $\mathcal{O}(h^2(\log h^{-1})^{d-1})$, which is only slightly worse than for the conventional, full-grid representation. In 1992, Griebel, Schneider and Zenger [2] showed that, for two and three dimensions, the sparse-grid complexity and representation error can also be achieved by the so-called combination technique. This technique combines $\mathcal{O}((\log h^{-1})^{d-1})$ representations on conventional grids of different mesh widths in different directions, each containing $\mathcal{O}(h^{-1})$ points into a representation on the conventional, full grid. In the current work, the asymptotic representation error for the combination technique is shown to be $\mathcal{O}(h^2 \log h^{-1})$, for the two-dimensional case. The current derivation does not involve the use of hierarchical basis functions. Instead, direct analyses are given of the steps that comprise the combination technique. Numerical results that confirm the analyses will be presented.

The work is directed towards the numerical solution of large-scale transport problems, governed by systems of partial differential equations of the advection-diffusion-reaction type. These equations play a prominent role in the mathematical modeling of pollution of, e.g, atmospheric air, surface water and ground water. The three-dimensional nature of these models and the necessity of modeling transport and chemical reactions between different species over long time spans, requires very efficient algorithms. When using full-grid methods, computer capacity (computing time and memory) is and will probably remain to be a severe limiting factor. Sparse-grid methods hold out the promise of alleviating these limitations.

1 C. Zenger, Sparse grids, in: W. Hackbusch, ed., *Notes on Numerical Fluid Mechanics*, **31**, 241–251 (Vieweg, Braunschweig, 1990).

2 M. Griebel, M. Schneider and C. Zenger, A combination technique for the solution of sparse grid problems, in: R. Beauwens and P. de Groen, eds., *Iterative Methods in Linear Algebra*, 263–281 (North-Holland, Amsterdam, 1992).

1 Presented by Boris Lastdrager

13.3 Multigrid Methods for Hierarchical Adaptive FE

Thu 17:00–17:30

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The hierarchical finite element method, using elements with variable aspect ratio, has proven to be useful for the numerical solution of PDEs and allows several concepts for the control of the adaptation process. In addition to the more conventional adaptive grids derived from L_2- or H^1-based error estimates, we study grids which are optimized with respect to the evaluation of linear functionals like the value of the solution at a fixed point. It is well-known that this requires the solution of a dual problem. As for the case of singular solutions, these grids are extremely refined at certain points, yielding different strategies for the solution vector and the right-hand side. This improves the order of the error with respect to L_2- or H^1-based adaptive grids, but causes additional difficulties for the design of efficient multigrid solvers.

1 Presented by Stefan Schneider

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