PHAML: A Parallel Adaptive Multilevel Program for Elliptic PDEs

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PHAML

• Parallel Hierarchical Adaptive Multi-Level
• Parallel sequel to MG GHAT
• 2D Elliptic PDE solver
• Fortran 90 module/library
• Adaptive finite elements with multigrid
• Message passing parallelism: MPI or PVM
• Tested on most UNIX systems
Elliptic Boundary Value Problems

\[-\frac{\partial}{\partial x} p \frac{\partial u}{\partial x} - \frac{\partial}{\partial y} q \frac{\partial u}{\partial y} + ru = f \text{ in } \Omega \subset \mathbb{R}^2 \]

\[u = g_1 \text{ on } \partial\Omega_1\]

\[\frac{\partial u}{\partial n} + cu = g_2 \text{ on } \partial\Omega_2\]

\(p, q, r, f, c, g_1\) and \(g_2\) functions of \((x, y)\)

\(\partial\Omega_1 \cup \partial\Omega_2 = \partial\Omega\)

\(\Omega\) polygonal

Also: time dependent, nonlinear, systems
Eigenvalue Problems

\[-\frac{\partial}{\partial x} p \frac{\partial u}{\partial x} - \frac{\partial}{\partial y} q \frac{\partial u}{\partial y} + ru = \lambda u \text{ in } \Omega \subset \mathbb{R}^2\]

\[u = 0 \text{ on } \partial \Omega_1\]

\[\frac{\partial u}{\partial n} = 0 \text{ on } \partial \Omega_2\]

\(p, q, r\) functions of \((x, y)\)

\(\partial \Omega_1 \cup \partial \Omega_2 = \partial \Omega\)

\(\Omega\) polygonal, generally a truncation of an infinite domain
Numerical Methods

- Parallelization of methods in MGGHAT
- Standard finite elements, linear, triangles
- Newest node bisection adaptive refinement
- Hierarchical basis multigrid
- Refinement–tree based grid partitioning
- Full domain partition
Optional Other Software

- BLAS and LAPACK
- MPI or PVM
- OpenGL, GLUT and F90GL
- PETSc
- MUMPS
- Zoltan, optionally including ParMETIS
- ARPACK

- More are likely to be added
  - Can provide a test bed for comparing programs/methods
Software Architecture

- Fortran 90 module
  - symbolic constants, data type and operators on it
- Compiles to a library
- User writes main program and subroutines that define the equation
- Two parallel paradigms
  - master/slave
  - SPMD
Primary Data Structure

- **type** `phaml_solution_type`
- Create, perform operations on it, and destroy
- Can have multiple objects
  - useful for time dependent, nonlinear, systems, etc.
- Contains all information about an object
  - grid and solution
  - processes information
    - consider the processes to be part of the object
    - spawned when created, terminated when destroyed
  - other state information and options
User Written External Routines

• Routines to define the problem to be solved
  • pdecoef
  • bcond
  • icond
  • init_grid (defines the domain)
  • true
  • a few others that are seldom used
Example pdecoef

subroutine pdecoef(x,y,p,q,r,f)

! pde is
! \(- (p(x,y)*u) - (q(x,y)*u) + r(x,y)*u = f(x,y)\)
! \( x \quad x \quad y \quad y \)

real, intent(in) :: x(:), y(:)
real, intent(out), optional :: p(:), q(:), r(:,), f(:)

if (present(p)) p = 1.0
if (present(q)) q = 1.0
if (present(r)) r = 0.0
if (present(f)) f = x**2 + y**2

end subroutine pdecoef
Callable Routines

- Operations that PHAML can perform on an object
  - phaml_create, phaml_destroy
  - phaml_solve_pde
  - phaml_evaluate
  - phaml_query
  - phaml_scale
  - phaml_integrate
  - phaml_connect
  - phaml_store, phaml_restore
  - phaml_popen, phaml_pclose
Optional Arguments

- Does not use a mysterious array to supply parameters
- Does not use a large workspace array that gets chopped up
- All options are specified in individual arguments
  - Nearly all arguments are optional
  - Missing arguments are given reasonable defaults
- Simplifies calling sequence
- Makes code more readable (keyword arguments)
- Improves upward compatibility
Arguments for `phaml_solve_pde`

```plaintext
subroutine solve_pde(phaml_solution, iterm, &
    max_elem, max_node, max_lev, maxRefsolveLoop, &
    init_form, comm_freq, partition_size, eq_type, &
    print_grid_when, print_grid_who, print_error_when, &
    print_error_who, print_time_when, print_time_who, &
    print_eval_when, print_eval_who, &
    print_header_who, print_trailer_who, clocks, &
    draw_grid_when, draw_refTree_when, pause_after_draw, &
    pause_after_phases, pause_at_start, pause_at_end, &
    uniform, overlap, sequential_node, inc_factor, &
    error_estimator, refTerm, derefine, &
    partition_method, predictive, &
    solver, preconditioner, mg_cycles, mg_prerelax, &
    mg_postrelax, iterations, ignore_quad_err, &
    final_solves, final_mg_cycles, &
    num_eval, lambda0)
```
Example main Program

program user_main_example
use phaml

type(phaml_solution_type) :: pde

call create(pde, draw_grid_who = MASTER)
call solve_pde(pde, &
    max_node = 20000, &
    draw_grid_when = PHASES, &
    partition_method = ZOLTAN_RCB, &
    mg_cycles = 2)

call destroy(pde)
end program
Conclusion

- PHAML is a new elliptic PDE solver
- Parallel sequel to MGGHAT
- Adaptive refinement, multigrid, message passing
- Hooks into several other software packages
- Tested on several unixes, compilers
- Now available
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